CHAPTER 12

GEAR TRAINS

12.1 GEAR TRAINS

In Chapters 10 and 11, the characteristics of individual gears were discussed. However, in general, gears are of interest to designers only when they are used in pairs as motion and/or force transducers. These gear pairs can be combined in many ways to achieve desired input–output relationships. A combination of one or more gear pairs that are interrelated is called a *gear train*. All complex gear trains are combinations of the simple, compound, and planetary gear trains discussed in this chapter.

12.2 DIRECTION OF ROTATION

As discussed in Chapters 10 and 11, the velocity ratio for two meshing gears (2 and 3) is

$$R = \pm\frac{\omega_2}{\omega_3} = \pm\frac{r_3}{r_2} = \pm\frac{N_3}{N_2} \qquad (12.1)$$

where r_i and N_i are the pitch radii and number of teeth on gear i ($i = 2, 3$), the plus sign goes with an external gear meshing with an internal gear, and the minus sign goes with two external gears meshing.

When planar gears are involved, Eq. (12.1) can be used directly because all of the vectors are parallel. However, when bevel and crossed helical gears are involved, the angular velocities must be treated as vectors. For bevel gears, a relatively simple way to do this is to recognize that at the pitch point (the end of the tangent line to the pitch surfaces), rolling occurs, and the velocity at the pitch point on both gears is the same. From this, the direction of rotation can be inferred from the simple velocity relationship

$$\mathbf{v}_{P_2} = \mathbf{v}_{P_3} = \boldsymbol{\omega}_2 \times \mathbf{r}_2 = \boldsymbol{\omega}_3 \times \mathbf{r}_3 \qquad (12.2)$$

where the pitch radius vectors ($\mathbf{r}_2$ and $\mathbf{r}_3$) for gears 2 and 3, respectively, are directed from the rotation axis to the pitch point. For bevel gears, the large end of the gear is used for the measurement of the pitch radii. This is shown in Fig. 12.1.

When crossed helical gears are involved, the process becomes a little more complicated because the hand of the gears, that is, the direction of twist of the helical teeth, affects the direction of rotation of the driven gear, as shown in Fig. 12.2. To determine the direction of rotation of the pinion relative to the gear, treat the pinion as a screw and the gear as fixed. Observe the motion of the pinion relative to the gear as the pinion is rotated and the gear is viewed along the gear axis. If the pinion appears to advance toward the gear when the pinion is rotated, in reality the gear would rotate counterclockwise. If the pinion appears to withdraw from the gear when the pinion is rotated, the gear would rotate clockwise.

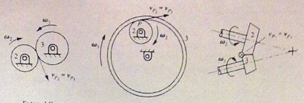


FIGURE 12.1 The direction of the angular velocities of two meshing gears can be determined from the direction of the velocity of the pitch point.

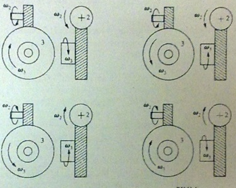


FIGURE 12.2 The direction of the angular velocities for crossed helical gears.

12.3 SIMPLE GEAR TRAINS

Simple gear trains can be divided into two types depending on whether idler gears are involved or not. Simple gear trains have only one gear on each shaft. These shafts rotate on bearings that are attached to the same frame. The gears may be of any type, for example, spur, bevel, hypoid, and worm. Figures 12.3–12.5 show various simple gear trains.

The idler gears in simple gear trains can serve two purposes in design. One is to change the direction of motion of the output gear, and the second is to provide a spacer when two gears cannot be directly meshed because of the shaft locations. This occurs when there is a limit to the sizes that two gears can be, but the shaft location is specified for reasons other than kinematics.

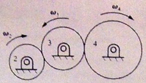


FIGURE 12.3 A simple gear train with all external gears and one idler (gear 3).

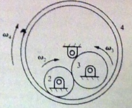


FIGURE 12.4 A simple gear train with internal gear and one idler (gear 3).

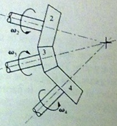


FIGURE 12.5 A simple gear train with external bevel gears and one idler (gear 3).

In gear trains, the overall gear reduction ratio for the gearbox is usually of interest. This can be determined by writing the velocity ratio in terms of the tooth numbers at each mesh. For example, in Fig. 12.3,

$$\frac{\omega_2}{\omega_3} = -\frac{N_3}{N_2} \tag{12.3}$$

and

$$\frac{\omega_3}{\omega_4} = -\frac{N_4}{N_3} \tag{12.4}$$

Multiplying Eq. (12.4) by Eq. (12.3) gives

$$\frac{\omega_2}{\omega_3}\frac{\omega_3}{\omega_4} = \left(-\frac{N_3}{N_2}\right)\left(-\frac{N_4}{N_3}\right)$$

or

$$\frac{\omega_2}{\omega_4} = (-1)^2 \frac{N_4}{N_2} \tag{12.5}$$

If we analyze the gear train in Fig. 12.4, we will get

$$\frac{\omega_2}{\omega_4} = (-1)^1 \frac{N_4}{N_2} \tag{12.6}$$

The analysis of the gear train in Fig. 12.5 is more difficult because we must treat the angular velocities as vectors to determine the directions mathematically. However, if we trace the angular-velocity directions using the procedure indicated in Fig. 12.1, we can determine the direction and compute the magnitude of the angular velocities separately. If we do this, we will find that the magnitude of the overall velocity ratio is given by

$$\frac{\omega_2}{\omega_4} = \frac{N_4}{N_2}$$

If we add more idler gears, the effect will be the same. Namely, the magnitude of the velocity ratio between the input and output shafts is a function of the numbers of teeth on the input and output gears only. The magnitude of the velocity ratio is independent of the size and number of idler gears. The sign of the train ratio for parallel-shaft gears, however, does depend on the number of idler gears. In particular, at each mesh between external gears, the velocity ratio changes sign. For internal gears, the velocity ratio remains the same sign. Therefore, if n is the number of meshes between *external* gears, the sign of the velocity ratio is given by $(-1)^n$. Note that each idler gear will have at least two mesh points.

A simple gear train can involve any number and types of gears. However, each gear in the gear train must be able to mesh with any other gear. Therefore, each gear must have the same normal pitch if the gears are to mesh properly.

12.3.1 Simple Reversing Mechanism

An idler gear can be used in a simple reversing mechanism shown in Fig. 12.6. This is a procedure commonly used to reverse the direction of rotation of the lead screw on small metal lathes. The procedure adds an extra idler to the simple gear train when the direction of rotation is to be reversed. The mechanism works well only when the gears are slowly moving or at rest, since there is no provision for ensuring that the gears will mesh easily when the direction change is made.

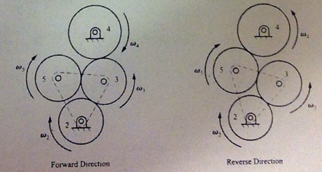


FIGURE 12.6 A simple reversing mechanism using an extra idler gear.

12.4 COMPOUND GEAR TRAINS

For all types of gears, the velocity ratio is limited for each mesh by practical considerations. For example, in spur gears, the velocity ratio at any mesh should not exceed 1:5. For larger reductions, compound gear trains should be used. Compound gear trains are characterized by the presence of two or more gears attached to the same shaft. The shafts, however, still rotate on bearings that are fixed to the frame. Unlike simple gear trains, the gears in a compound gear train need not and generally will not be of the same type. This is evident in Fig. 12.7, which shows an example of a commercial gear reducer.

The velocity ratios attainable in a compound gear train can be any size, with ratios in the thousands being possible. There is no theoretical limit to the number of passes (gear meshes) that can be made; however, practical issues such as friction and the functional need restrict the number in most applications.

A compound gear train is shown in Fig. 12.8. The symbolism often used for gear trains is also illustrated in the figure. The velocity ratio for the gear mesh can be written as

$$\frac{\omega_2}{\omega_3} = -\frac{N_3}{N_2} \tag{12.7}$$

and

$$\frac{\omega_4}{\omega_5} = -\frac{N_5}{N_4} \tag{12.8}$$

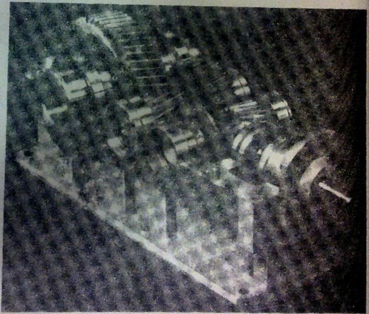


FIGURE 12.7 A compound gear train. (Courtesy of PT Components, Inc., Indianapolis, Indiana.)

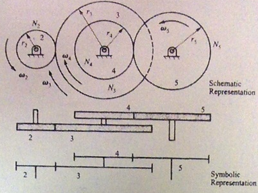


FIGURE 12.8 A compound gear train.

Because gears 3 and 4 are rigidly attached to the same shaft, we know that $\omega_3 = \omega_4$. The overall gear train velocity ratio is given by ω_2/ω_5.

If we solve Eq. (12.7) for ω_2 and Eq. (12.8) for ω_5 and use $\omega_3 = \omega_4$, the overall velocity ratio can be written as

$$\frac{\omega_2}{\omega_5} = \frac{-\omega_3\left(\dfrac{N_3}{N_2}\right)}{-\omega_4\left(\dfrac{N_4}{N_5}\right)} = \frac{\left(\dfrac{N_3}{N_2}\right)}{\left(\dfrac{N_4}{N_5}\right)} = \frac{N_3 N_5}{N_2 N_4} \tag{12.9}$$

or

$$\frac{\omega_2}{\omega_5} = \frac{\omega_2}{\omega_3}\frac{\omega_4}{\omega_5} = \frac{N_3 N_5}{N_2 N_4} \tag{12.10}$$

From Eq. (12.10), it is clear that we can compute the overall velocity ratio from the product of the velocity ratios at each mesh. We can do this either in terms of the velocities directly, or more beneficially, in terms of the tooth numbers. Notice that when parallel-shaft gearing is involved, we can also use the ratios of the pitch-circle radii, because these will be directly proportional to the tooth numbers. Equation (12.10) can then be extended to

$$\frac{\omega_2}{\omega_5} = \frac{\omega_2}{\omega_3}\frac{\omega_4}{\omega_5} = \frac{N_3 N_5}{N_2 N_4} = \frac{r_3 r_5}{r_2 r_4} \tag{12.11}$$

However, the most convenient parameter to use when computing velocity ratios is the tooth number on each gear. This is because the velocity ratios can be directly equated to the tooth ratios for all types of gearing, whereas the ratios of the pitch-cylinder radii alone are not valid for gears with nonparallel shafts.

If we start with gear 2 as the input gear, we can treat each mesh of the gear train as having an input side and an output side. For example, in Fig. 12.8, at the mesh between gears 2 and 3, gear 2 would be the driver and gear 3 would be the driven gear. At the mesh between gears 4 and 5, gear 4 would be the driver and gear 5 would be the driven gear. Therefore, N_3 and N_5 would be associated with driven gears, and N_2 and N_4 would be associated with driver gears. In Eq. (12.11), the velocity ratio can be represented as the product of the driven gear tooth numbers divided by the product of the driver gear tooth numbers. This situation holds in general for compound gear trains. Mathematically, if n is the number of gear meshes (including idlers that each have two meshes), a general expression for the magnitude of the velocity ratio can be written as

$$\frac{\omega_{input}}{\omega_{output}} = \frac{\prod_{i=3}^{n} N_i}{\prod_{j=3}^{n} N_j} = \frac{\text{product of driven tooth numbers}}{\text{product of driver tooth numbers}} \tag{12.12}$$

Assuming that the gears are numbered sequentially, in Eq. (12.12), i includes only the tooth numbers for the odd gear numbers, and j includes only the tooth numbers for the even gear numbers. The sign of the gear ratio depends on the type of gears. If all parallel-shaft gears are involved, we can use an extension of Eq. (12.4). Then,

$$\frac{\omega_{input}}{\omega_{output}} = (-1)^m \frac{\prod_{i=3}^{n} N_i}{\prod_{j=3}^{n} N_j} = (-1)^m \frac{\text{product of driven tooth numbers}}{\text{product of driver tooth numbers}}$$

where m is the number of meshes involving external gears.

EXAMPLE 12.1
Analysis of a Compound Gear Train

Assume that the compound gear train in Fig. 12.9 has the tooth numbers given in parentheses. The angular velocity of gear 2 is 200 rpm in the direction shown. Find the magnitude and direction of the angular velocity of gear 10 and the velocity (magnitude and direction) of the rack that is gear 11.

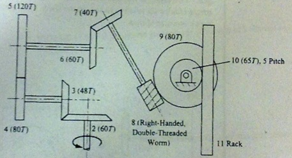


FIGURE 12.9 The gear train for Example 12.1.

Solution

The velocity ratio for the gear drive between gears 2 and 9 is given by

$$\frac{\omega_9}{\omega_2} = \frac{N_2}{N_3}\frac{N_4}{N_5}\frac{N_6}{N_7}\frac{N_8}{N_9}$$

Therefore,

$$\omega_9 = \omega_{10} = \omega_2 \frac{N_2}{N_3}\frac{N_4}{N_5}\frac{N_6}{N_7}\frac{N_8}{N_9}$$

and

$$\omega_9 = \omega_{10} = 200 \frac{60}{48}\frac{80}{120}\frac{60}{40}\frac{2}{80} = 6.25 \text{ rpm}$$

The velocity of the rack will be equal to the linear velocity of the pitch point on gear 10. The angular velocity of gear 10 is equal to the angular velocity of gear 9. The pitch diameter of gear 10 is given by

$$d_{10} = \frac{N_{10}}{P_{10}} = \frac{65}{5} = 13 \text{ in}$$

Therefore, the pitch velocity is given by

$$v = (\omega_{10})(d_{10}/2) = (6.25)(13/2)(2\pi/60) = 4.255 \text{ in/s}$$

However, we must now determine in which direction the rack moves (up or down). To do this, trace the pitch point velocities at each mesh. This is shown in Fig. 12.10. Gear 8 is a right-handed worm gear. Therefore, it will advance relative to gear 9 for a clockwise rotation. Consequently, gear 9 will rotate clockwise relative to the frame (link 1). If gear 9 and 10 rotate clockwise, then the rack will move down as shown.

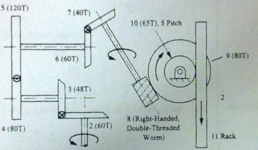


FIGURE 12.10 The directions of gear motion for Example 12.1.

12.4.1 Concentric Gear Trains

In a concentric gear train, the input and output shafts are collinear. An example of a concentric gear train is shown in Fig. 12.11. These gear trains are analyzed in much the same way as any compound gear train; however, the design is somewhat more complex. In the following, we will restrict the discussion to parallel-shaft gearing with a double reduction to illustrate a possible design procedure.

A concentric gear train with a two-stage reduction is shown in Fig. 12.12. A principal requirement for a concentric gear reducer is that

$$r_2 + r_3 = r_4 + r_5 \tag{12.13}$$

In addition, there may be a precise requirement for the overall reduction ratio. From before, the reduction ratio is given by

$$R = \frac{\omega_2}{\omega_5} = \frac{r_3}{r_2}\frac{r_5}{r_4} = \frac{N_3}{N_2}\frac{N_5}{N_4} \tag{12.14}$$

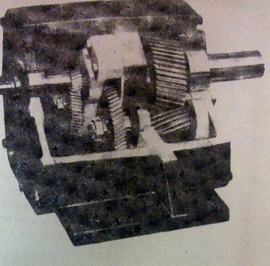


FIGURE 12.11 A concentric gear reducer. (Courtesy of Rockwell Automation Dodge, Greenville, South Carolina.)

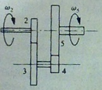


FIGURE 12.12 A concentric gear reducer with a two-stage reduction.

To mesh properly, gears 2 and 3 must have the same normal pitch, and gears 4 and 5 must have the same normal pitch. If helical gears are involved, we must select the helix angles, and this will give us some latitude in the design. Then,

$$\frac{N_2}{2r_2} = P_{n_2} \cos\psi_2$$
$$\frac{N_3}{2r_3} = P_{n_2} \cos\psi_2 \qquad (12.15)$$

and

$$\frac{N_4}{2r_4} = P_{n_4} \cos\psi_4$$
$$\frac{N_5}{2r_5} = P_{n_4} \cos\psi_4 \qquad (12.16)$$

In Eqs. (12.13)–(12.16) there are 12 unknowns and 6 equations. In addition, there is the constraint that the tooth numbers must be integers. Therefore, we can select six of the variables to solve the equations subject to the constraint that the tooth numbers are integers.

One design approach is to select first the tooth numbers to satisfy Eq. (12.14), which is typically the most difficult equation to satisfy. This is equivalent to selecting three of the variables. It may not always be easy or even possible to select tooth numbers in a practical range to solve Eq. (12.14) exactly. If the values of R are formed by ratios of small whole numbers, for example 1/2, 7/4, or 4/9, many choices of whole numbers will satisfy the problem. In such cases, the best choice can be selected on the basis of criteria other than kinematics. However, other values of R are impossible to generate with simple gears. Examples are the square root of 2 and the ratio of two prime numbers (e.g., 503/2003). In such cases, it may only be possible to approximate the value for R.

When the machine function does not require an exact ratio, it is usual to select tooth numbers for a meshing gear pair that do not have common factors. This improves wear performance, because a defect on a gear tooth of one gear will make contact with all of the teeth on the mating gear equally rather than selectively making contact with a small number of teeth.

A number of elegant procedures are available for selecting the tooth numbers given R especially when R is given as a fraction where the numerator and denominator are whole numbers.[1] Such procedures may be aided by tables of factors. For example, if $R = p/q$, we would look for values of N_1 and N_5 such that $N_3N_5 = p$ and values of N_2 and N_4 such that $N_2N_4 = q$. Alternatively, if the ranges for the tooth numbers are limited, we might simply conduct an exhaustive search for all possible combinations of tooth numbers that satisfy the condition for R. On modern computers, such a search is easy to program and takes very little time to conduct. A simple MATLAB program, *factor.m*, for finding factors of any integer is included on the disk with this book.

After the tooth numbers are established, we can select one of the normal diametral pitches, for example, P_{n_2}, and the corresponding helix angle (ψ_2). Then solve for r_2 and r_3. Given r_2 and r_3, Eqs. (12.13) and (12.16) can be solved for r_4, r_5, and $P_{n_4}\cos\psi_4$. Pick a standard value for P_{n_4}, and solve for the helix angle, ψ_4.

This discussion deals with kinematics alone. Obviously, other very important aspects of gear design are stress and wear considerations. The topic is properly treated in almost any book on machine design. Therefore, we will limit our discussion to kinematics with the assumption that sizing the teeth to carry the loading will be addressed elsewhere.

EXAMPLE 12.2
Concentric Gearbox Design

Assume that a concentric gearbox is to be designed for a velocity ratio of $R = 20{:}1$. The first stage reduction is to have a helix angle of 30° and normal diametral pitch of 8. Both sets of gears will have a normal pressure angle of 20°. Find values for the tooth numbers, pitch cylinder radii for all of the gears, and the diametral pitch and helix angle for gears 4 and 5.

Solution

To avoid undercutting, we will limit the tooth numbers for N_2 and N_3 to 12 teeth (see Table 11.1). For the smallest possible gear box, assume that N_2 is 12. Initially, select N_4 to be 12 also. If the minimum helix angle for gears 4 and 5 is found to be less than 30°, we can specify it to be 30°, which will make 12 an acceptable tooth number. If the helix angle must be larger than 30°, the minimum value for N_4 could be smaller than 12. The velocity ratio can be written as

$$R = \frac{20n}{n}$$

where n is any integer. From Eq. (12.14), $n = N_2N_4$. Therefore, $n = 144$, and $20n$ is 2880.

To determine the factors for 2880, the MATLAB factor program was used. There are 42 factors for 2880. These are 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 30, 32, 36, 40, 45, 48, 60, 64, 72, 80, 90, 96, 120, 144, 160, 180, 192, 240, 288, 320, 360, 480, 576, 720, 960, 1440, and 2880. When designing the two-stage gear reducer, it is generally desirable to make the two gear reductions about the same. This avoids making one gear significantly larger than the others. Of the factors, 48 and 60 will give gear reductions of 4 and 5 for the two stages. Let us select 60 for the first stage (N_3) and 48 for the second stage (N_3). This will permit larger teeth (lower P_n) on the low-speed end of the gear reducer without making the gear diameters significantly larger than those for the high-end gears.

From Eqs. (12.15),

$$r_2 = \frac{N_2}{2P_{n_2}\cos\psi_2} = \frac{12}{2(8)\cos 30^\circ} = 0.866 \text{ in}$$

and

$$r_3 = \frac{N_3}{2P_{n_3}\cos\psi_3} = \frac{N_3}{2P_{n_2}\cos\psi_2} = \frac{60}{2(8)\cos 30^\circ} = 4.330 \text{ in}$$

From Eqs. (12.16),

$$\frac{N_4}{N_5} = \frac{r_4}{r_5} = \frac{12}{48} = \frac{1}{4}$$

Therefore,

$$r_5 = 4r_4$$

Substituting this expression into Eq. (12.13) gives

$$5r_4 = r_2 + r_3 = 0.866 + 4.330 = 5.196 \text{ in}$$

Therefore,

$$r_4 = \frac{5.196}{5} = 1.039 \text{ in}$$

and

$$r_5 = 4r_4 = 4(1.039) = 4.156 \text{ in}$$

The only unknowns are P_{n_4} and ψ_4. From Eqs. (12.16),

$$P_{n_4}\cos\psi_4 = \frac{N_4}{2r_4} = \frac{12}{2(1.039)} = 5.775$$

To illustrate the procedure, select a normal diametral pitch of 7. Then

$$\psi_4 = \cos^{-1}\left(\frac{5.775}{7}\right) = \cos^{-1}(0.825) = 34.4^\circ$$

Note that in this example we have arbitrarily selected the diametral pitches. In an actual problem, these, along with the face widths, would be selected in part to accommodate the torque and speed requirements.

12.5 PLANETARY GEAR TRAINS

Both simple and compound gear trains have the restriction that their gear shafts must rotate in bearings fixed to the frame. However, this is a requirement that limits the versatility of the gear train. If one or more shafts rotate around another shaft as well as spinning about their own axes, the gear train is called a *planetary* or *epicyclic gear train*. Planetary gear trains are used extensively for compact gear reducers (Fig. 12.13). Also, because they are

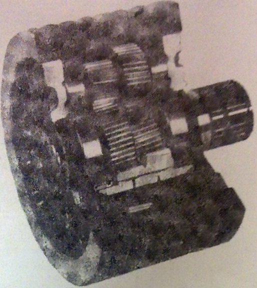


FIGURE 12.13 A planetary gear reducer. (Courtesy of Andantex U.S.A., Wanamassa, New Jersey.)

basically devices with multiple degrees of freedom. they are used in automatic transmissions for automobiles and trucks (Fig. 12.14).

Determining the velocity ratio for planetary gear trains is more difficult than for plain simple and compound gear trains. The motion of the gears involves the motion of the moving shaft or carrier along with the motion of the gears with respect to the carrier.

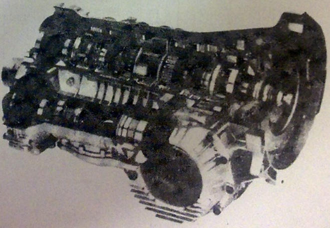


FIGURE 12.14 Planetary gears in an automatic transmission. (Courtesy of Zahnradfabrik Passau GmbH, Germany.)

12.5.1 Planetary Gear Nomenclature

A simple planetary drive is shown schematically in Fig. 12.15. Planetary gear trains are typically made up of the following:

1. a sun gear (which may or may not be fixed),
2. planet gears (one or more),
3. a planet carrier, and
4. an internal ring gear (not used in all planetary gear trains).

The symbolism used to represent planetary drives is also shown in Fig. 12.15. This symbolism allows the designer to represent the structure of the planetary drive simply.

In Fig. 12.15, note that the carrier, ring gear, and sun gear all rotate about concentric axes. Also, three axes are evident from the simple figure. As shown in the following, the planetary gear train has two degrees of freedom, and the angular motion of two of the axes must be specified before the angular motion of the third can be determined.

Very high velocity reductions can be achieved with compound planetary gear trains. These systems involve compound planetary gears as shown in Fig. 12.16a. This also permits the ring gear in Fig. 12.15 to be replaced by another sun gear. The carrier can involve several shafts containing four or more planetary gears as shown in Fig. 12.16b for still greater reductions. And finally, it is possible to connect planetary gears in series as shown in Fig. 12.16c. In Fig. 12.16, the bearings associated with the frame link are not shown. It is understood that frame bearings will be required for all of the shafts rotating with fixed axes.

In the planetary gear trains indicated in Fig. 12.16, two of the shafts are inputs and one is the output. Typically, the angular velocity of one of the bodies is zero, but this is not required.

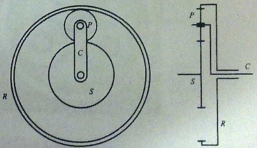


FIGURE 12.15 A simple planetary gear train.

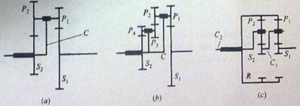


FIGURE 12.16 Complex planetary gear trains.

When planetary gear trains are connected in series, very high gear reductions are possible. Such gear reducers are common in small power tools. Figure 12.17 shows a small planetary gear reducer made up of two planetary systems in series used in a power screwdriver.

In Figs. 12.13–12.17, all of the gears in the planetary drives are parallel-shaft gears. However, this is not necessary. Perhaps the most common planetary drive is the differential (Fig. 12.18) in automobiles. This is a right-angle drive that involves a hypoid ring gear and pinion and bevel gear planets.

Planetary gear trains are commonly analyzed using either the equation method or the tabular method. We will look at each procedure separately by analyzing example gear trains.

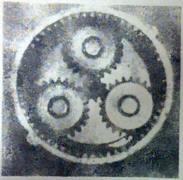


FIGURE 12.17 Planetary gear trains used in a power screwdriver.

FIGURE 12.18 A planetary gear train used in the differential of a rear-wheel-driven vehicle. (Courtesy of The Gleason Works, Rochester, NY.)

12.5.2 Analysis of Planetary Gear Trains Using Equations

In the equation method, the procedure is to write relative angular velocity equations (relative to the frame) for each of the gears with fixed rotation axes. Also, write relative velocity equations for the same gears relative to the carrier. If the angular velocities of two of the shafts are given, this procedure will always yield enough equations to solve for the angular velocities of all of the members in the system.

EXAMPLE 12.3 Basic Procedure Using the Equation Method

Assume that the gear train in Fig. 12.15 has the frame as member 1, the sun gear as member 2, the planet as member 3, and the ring gear as member 4. Gear 2 is the input and rotates CW with an angular velocity of ω_2. Find an equation that involves the angular velocity of the carrier, ω_2, ω_4, and the tooth numbers for the individual gears.

Solution

There are two gears (2 and 4) that rotate about fixed axes in the system, and we can write the following angular velocity relationships for these gears using the chain rule for angular velocities:

$$\omega_2 = {}^{C}\omega_2 + \omega_C \tag{12.17}$$

and

$$\omega_4 = {}^{C}\omega_4 + \omega_C \tag{12.18}$$

If we make the carrier the reference link, the gears will move as an ordinary gear train in which the planet gear acts as an idler. Therefore, we can compute the velocity ratio relative to the carrier as

$$\frac{{}^{C}\omega_2}{{}^{C}\omega_4} = -\frac{N_4}{N_2} \tag{12.19}$$

Next solve Eqs. (12.17) and (12.18) for ${}^{C}\omega_2$ and ${}^{C}\omega_4$, respectively. Then

$${}^{C}\omega_2 = \omega_2 - \omega_C \tag{12.20}$$

and

$${}^{C}\omega_4 = \omega_4 - \omega_C \tag{12.21}$$

Now divide Eq. (12.20) by (12.21) and equate the result to Eq. (12.19). The result is

$$\frac{\omega_2 - \omega_C}{\omega_4 - \omega_C} = -\frac{N_4}{N_2} \tag{12.22}$$

Equation (12.22) gives the relationship for the velocities of the shafts coming from the gear train. Given any two of the angular velocities, the third can be determined. Note that it is important to identify the direction of the angular velocities with a plus or minus sign. Typically, we could select CCW as plus and CW as minus.

EXAMPLE 12.4 Analysis of a Planetary Gear Train Using the Equation Method

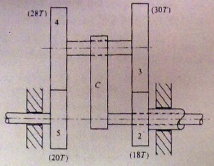


FIGURE 12.19 The planetary gear train for Example 12.4.

Solution There are two gears (2 and 5) that rotate about fixed axes in the system. As in the case of the previous example, we can write the angular velocity relationships for these gears using the chain rule for angular velocities. The equations are

$$\omega_2 = {}^C\omega_2 + \omega_C \tag{12.23}$$

and

$$\omega_5 = {}^C\omega_5 + \omega_C \tag{12.24}$$

The angular velocity ratio of gears 2 and 5 relative to the carrier is

$$\frac{{}^C\omega_2}{{}^C\omega_5} = \frac{N_5}{N_4}\frac{N_3}{N_2} \tag{12.25}$$

Notice that the velocity ratio is positive because both gears rotate in the same direction relative to the arm. Next solve Eqs. (12.23) and (12.24) for ${}^C\omega_2$ and ${}^C\omega_5$, respectively. Then,

$${}^C\omega_2 = \omega_2 - \omega_C \tag{12.26}$$

and

$${}^C\omega_5 = \omega_5 - \omega_C \tag{12.27}$$

Now divide Eq. (12.26) by (12.27) and equate the result to Eq. (12.25). This gives

$$\frac{\omega_2 - \omega_C}{\omega_5 - \omega_C} = \frac{N_5}{N_4}\frac{N_3}{N_2} \tag{12.28}$$

Assuming CCW as positive, from the problem statement, $\omega_C = -150$ rpm and $\omega_4 = -50$ rpm. The tooth numbers are given in Fig. 12.19. Substituting the known values into Eq. (12.28) gives

$$\frac{\omega_2 + 150}{-50 + 150} = \frac{20}{28}\frac{30}{18}$$

or

$$\omega_2 + 150 = 119.04$$

or

$$\omega_2 = -30.95 \text{ rpm}$$

Therefore, the velocity of gear 2 is 30.95 rpm in the CW direction.

EXAMPLE 12.5
Analysis of a Planetary Gear Train Using the Equation Method

Assume that gear 2 in Fig. 12.20 is driven at a speed of 60 rpm in the CCW direction viewed from the right end. Gear 4 meshes with a fixed ring gear and with gear 5 as shown. Find the magnitude and direction of the angular velocity of gear 5.

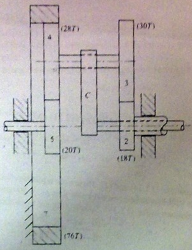


FIGURE 12.20 The planetary gear train for Example 12.5.

Solution

There are three gears (2, 5, and 7) that can rotate about fixed axes in the system. We include gear 7 in this list but ultimately will use the fact that its velocity is zero. As in Examples 12.3 and 12.4, we will solve the problem by writing relative velocity equations for all of the gears that have shafts that can rotate in fixed bearings. After rearranging, we have the resulting equations

$$^c\omega_2 = \omega_2 - \omega_c \tag{12.29}$$

$$^c\omega_5 = \omega_5 - \omega_c \tag{12.30}$$

and

$$^c\omega_7 = \omega_7 - \omega_c \tag{12.31}$$

The angular velocity ratio of gears 2 and 5 relative to the carrier is

$$\frac{^c\omega_2}{^c\omega_5} = \frac{N_5}{N_4}\frac{N_3}{N_2} \tag{12.32}$$

and that of gears 2 and 7 relative to the carrier is

$$\frac{^c\omega_2}{^c\omega_7} = -\frac{N_7}{N_4}\frac{N_3}{N_2} \tag{12.33}$$

Now, divide Eq. (12.29) by Eq. (12.30) and equate the result with Eq. (12.32). This gives

$$\frac{\omega_2 - \omega_c}{\omega_5 - \omega_c} = \frac{N_5}{N_4}\frac{N_3}{N_2} \tag{12.34}$$

Similarly, divide Eq. (12.29) by (12.31) and equate the result with Eq. (12.33) to get

$$\frac{\omega_2 - \omega_C}{\omega_7 - \omega_C} = -\frac{N_7}{N_4}\frac{N_3}{N_2} \tag{12.35}$$

Equations (12.34) and (12.35) are the equations necessary for analyzing the planetary gear train. From the problem statement, we know that $\omega_2 = 60$ rpm and $\omega_7 = 0$. With these known values, only ω_C is unknown in Eq. (12.35). Substituting the known values into Eq. (12.35) gives

$$\frac{60 - \omega_C}{0 - \omega_C} = -\frac{76}{28}\frac{30}{18} = -4.5238$$

Solving gives

$$\omega_C(1 + 4.5238) = 60 \Rightarrow \omega_C = 10.862 \text{ rpm}$$

Given ω_C and ω_2, we can solve Eq. (12.34) for ω_5. Substituting the known values into Eq. 12.34) gives

$$\frac{60 - 10.862}{\omega_5 - 10.862} = \frac{20}{28}\frac{30}{18} = 1.1905$$

Solving for ω_5 yields

$$\omega_5 - 10.862 = \frac{60 - 10.862}{1.1905} = 41.275 \Rightarrow \omega_5 = 52.137 \text{ rpm}$$

The value is positive, so ω_5 is rotating CCW when viewed from the right.

EXAMPLE 12.6
Analysis of Planetary Gear Trains in Series

A two-stage planetary gear drive is represented in Fig. 12.21. Gear 2 is the input member, and carrier 7 is the output member. Gear 4 is a ring gear and is fixed. The carrier of the first stage is member 6, and it is rigidly connected to the gear that drives the second stage. Determine the velocity ratio of the gear drive.

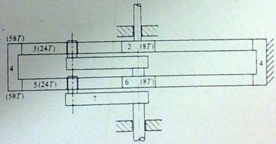


FIGURE 12.21 The planetary gear train for Example 12.6.

Solution

There are three gears (2, 4, and 6) that can rotate about fixed axes in the system. Again, we will include the fixed ring gear in the equations and will set the velocity to zero once the equations are developed. As in the previous examples, we will solve the problem by writing relative velocity equations for all of the gears that have shafts that can rotate in fixed bearings. However, we must separate the two stages of the planetary drives when we write the equations. The first stage includes gears 2, 3, and 4 and the carrier is member 6. The second stage includes gears 6, 5, and 4 and carrier 7. The first stage can be analyzed independently of the second stage to determine the velocity of gear 6 in terms of the velocity of gear 2. The second stage can then be analyzed to determine the velocity of the second carrier (7) in terms of the velocity of gear 6. By combining the results of both stages, the velocity of link 7 can be determined as a function of gear 2 to determine the overall velocity ratio of the gear train.

After being rearranged, the first-stage relative velocity equations are

$$ {}^{6}\omega_2 = \omega_2 - \omega_6 \tag{12.36} $$

$$ {}^{6}\omega_4 = \omega_4 - \omega_6 \tag{12.37} $$

The angular velocity ratio of gears 2 and 4 relative to the carrier (member 6) is

$$ \frac{{}^{6}\omega_2}{{}^{6}\omega_4} = -\frac{N_4}{N_2} \tag{12.38} $$

Now, divide Eq. (12.36) by Eq. (12.37) and equate the result with Eq. (12.38). This gives

$$ \frac{\omega_2 - \omega_6}{\omega_4 - \omega_6} = -\frac{N_4}{N_2} \tag{12.39} $$

Because $\omega_4 = 0$, this equation can be rewritten for ω_6 as a function of ω_2. The result is

$$ \omega_6 = \frac{\omega_2}{1 + N_4/N_2} \tag{12.40} $$

We can now analyze the second stage in exactly the same manner as the first stage except that now the gears are 6, 5, and 4, and the carrier is 7. After being rearranged, the second-stage relative velocity equations are

$$ {}^{7}\omega_6 = \omega_6 - \omega_7 \tag{12.41} $$

$$ {}^{7}\omega_4 = \omega_4 - \omega_7 \tag{12.42} $$

The angular velocity ratio of gears 4 and 6 relative to the carrier (member 7) is

$$ \frac{{}^{7}\omega_6}{{}^{7}\omega_4} = -\frac{N_4}{N_6} \tag{12.43} $$

Now, divide Eq. (12.41) by Eq. (12.42) and equate the result with Eq. (12.43). This gives

$$ \frac{\omega_6 - \omega_7}{\omega_4 - \omega_7} = -\frac{N_4}{N_6} \tag{12.44} $$

Because $\omega_4 = 0$, this equation can be rewritten for ω_7 as a function of ω_6. The result is

$$ \omega_7 = \frac{\omega_6}{1 + N_4/N_6} \tag{12.45} $$

Combining Eqs. (12.45) and (12.40) and substituting the known tooth numbers gives

$$ \frac{\omega_2}{\omega_7} = \left(1 + \frac{N_4}{N_2}\right)\left(1 + \frac{N_4}{N_6}\right) = \left(1 + \frac{58}{8}\right)\left(1 + \frac{58}{8}\right) = 68.06 $$

Notice that, in this example, the size of the planet gears does not affect the velocity ratio. They will have an impact on the size of the gearbox, however. Also notice that members 2 and 7 both rotate in the same direction.

EXAMPLE 12.7. Analysis of a Planetary Gear Train with Bevel Gears

All of the planetary gear trains considered in the previous examples involved only parallel-shaft gears. In this example, we will analyze a gear train with bevel gears. The gear train is shown in Fig. 12.22. The input to the gear train is gear 2 and the output is gear 6. The carrier is member 3, which rotates freely about the shaft on gear 6. Also, the compound planet gears (4 and 5) rotate about the axis of the carrier. Gear 7 is fixed to the frame. Assume that gear 2 rotates with an angular velocity of 100 rpm in the CCW direction viewed from the right. Find the angular velocity of gear 6.

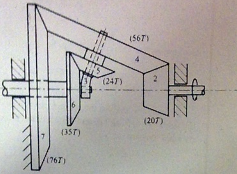


FIGURE 12.22 The planetary gear train for Example 12.7.

Solution

Even though bevel gears are involved, we can approach the analysis in exactly the same way that was used for the parallel-shaft gears. There are three gears (2, 6, and 7) that can rotate about fixed axes in the system. Again, we will include gear 7 in this list but ultimately will use the fact that its velocity is zero. As in the previous examples, we will solve the problem by writing relative velocity equations for all of the gears that have shafts that can rotate in fixed bearings. After being rearranged, the resulting equations are

$$^3\omega_2 = \omega_2 - \omega_3 \tag{12.46}$$

$$^3\omega_6 = \omega_6 - \omega_3 \tag{12.47}$$

and

$$^3\omega_7 = \omega_7 - \omega_3 \tag{12.48}$$

The angular velocity ratio of gears 2 and 6 relative to the carrier (3) is

$$\frac{^3\omega_2}{^3\omega_6} = -\frac{N_4}{N_2}\frac{N_6}{N_5} \tag{12.49}$$

In Eq. (12.49) we must determine the direction by inspection. This will show that if the carrier is fixed and the motion is inverted so that all of the other links and gears (including 4) can move relative to the carrier, gears 2 and 6 will move in opposite directions. Similarly, the motion of gears 2 and 7 relative to the carrier is

$$\frac{^3\omega_2}{^3\omega_7} = -\frac{N_7}{N_2} \tag{12.50}$$

Again, relative to the carrier, gears 2 and 7 are seen to move in opposite directions. Now, divide Eq. (12.46) by Eq. (12.48) and equate the result with Eq. (12.50). This gives

$$\frac{\omega_2 - \omega_3}{\omega_6 - \omega_3} = -\frac{N_4}{N_2}\frac{N_6}{N_5} \tag{12.51}$$

Also divide Eq. (12.46) by Eq. (12.47) and equate the result to Eq. (12.49). This gives

$$\frac{\omega_2 - \omega_3}{\omega_7 - \omega_3} = -\frac{N_7}{N_2} \tag{12.52}$$

Equations (12.51) and (12.52) are the equations necessary for analyzing the planetary gear train. From the problem statement, we know that $\omega_2 = 100$ and $\omega_7 = 0$. With these known values, only ω_3 is unknown in Eq. (12.52). Substituting the known values into Eq. (12.52) gives

$$\frac{100 - \omega_3}{0 - \omega_3} = -\frac{76}{20} = -3.8$$

Solving gives

$$\omega_3(1 + 3.8) = 100 \Rightarrow \omega_3 = 20.833 \text{ rpm}$$

Given ω_3 and ω_2, we can solve Eq. (12.51) for ω_6. Substituting the known values into Eq. 12.51) gives

$$\frac{100 - 20.833}{\omega_6 - 20.833} = -\frac{56}{20}\frac{35}{24} = -4.0833$$

Solving for ω_6 gives

$$\omega_6 = \frac{100 - 20.833}{-4.0833} + 20.833 = 1.444 \text{ rpm}$$

The value is positive, so ω_6 is rotating in the same direction as ω_2. Therefore, ω_6 is rotating when viewed from the right. The overall velocity ratio for the gearbox is

$$\frac{\omega_2}{\omega_6} = \frac{100}{1.444} = 69.2$$

12.5.3 Analysis of Planetary Gear Trains Using the Tabular Method

Overview The tabulation method is based on the knowledge that a planetary gear train is a linear system. The absolute angular velocity of any gear x that rotates about an axis fixed to the frame can be written as

$$\omega_x = {}^C\omega_x + \omega_C \tag{12.53}$$

where ω_C is the absolute angular velocity of the carrier and ${}^C\omega_x$ is the angular velocity of the gear relative to the carrier. Also, because Eq. (12.53) is linear, we can multiply the input values by a constant, and the output value will be multiplied by the same constant. The tabular method is based on the idea of the linear relationship shown in Eq. (12.53) and superposition.

A simple planetary gear train is fundamentally a two-degree-of-freedom device. Therefore, we must specify two input velocities or displacements to compute the unknown velocity or displacement. For discussion, consider the planetary gear train shown in Fig. 12.23. There is a compound gear for the planets and two sun gears, and the members are each assigned a number. Assume that the carrier is one of the input members, gear 2 is the other input member, and the known velocities are ω_2 and ω_5, respectively.

Now let us fix all of the gears to the carrier and rotate all of the gears by the velocity ω_5. Then all of the gears, including gear 2, will have an initial angular velocity of ω_5. To correct the angular velocity of gear 2 without changing the angular velocity of the carrier, let us fix the angular velocity of the carrier and move all of the gears relative to the carrier such that angular velocity of gear 2 ends up with the correct value when added to the velocity (ω_5) from step 1. This will also change all of the angular velocities in the gear train. The vector sum of the angular velocities from steps 1 and 2 for each gear will give the correct values. The vector sum is simply an algebraic sum because all of the gears have parallel shafts. The procedure, shown schematically in Fig. 12.23, is formalized in the tabular method of analysis as follows.

Procedure The tabulation method begins with a table in which there is one column for each member in the gear train and a row for each of the following three steps.

1. Assume that all of the gears are locked to the carrier, and rotate the assembly with an angular velocity equal to the angular velocity of the carrier member. Tabulate this velocity under each member in the train.
2. Fix the arm, and rotate the second input member such that it ends up with the proper input velocity when steps 1 and 2 are added together. Tabulate the resulting velocity for each member in the train.
3. Add the results from steps 1 and 2 for each member in the train.

We will illustrate the procedure on three examples.

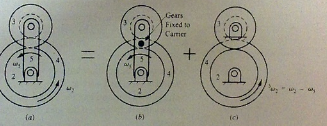


FIGURE 12.23 Analysis of a planetary drive using superposition.

EXAMPLE 12.8
Tabulation Method for a Simple Planetary Gear Train

Assume that the planetary gear train in Fig. 12.24 has the frame as member 1, the sun gear as member 2, the planet gear as member 3, the ring gear as member 4, and the carrier as member 5. Gear 2 is the input and rotates CCW with an angular velocity of 100 rpm and the carrier rotates CCW with an angular velocity of 200 rpm. Find the angular velocities of gears 3 and 4.

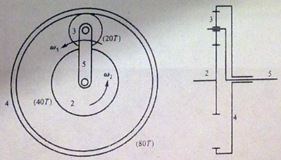


FIGURE 12.24 The planetary gear train for Example 12.8.

Solution

Following the aforementioned procedure, we can construct the solution table (Table 12.1). The four members of the system are associated with the columns, and the steps are associated with the rows. Step 1 of the procedure is to rotate the entire assembly by + 200 rpm, which is the velocity of the carrier. Next, we will fix the carrier and rotate gear 2 by the velocity required to make its total angular velocity 100 rpm. To do this, we need to rotate gear 2 by − 100 rpm relative to the arm.

TABLE 12.1 Results for Example 12.8

Step	Carrier 5 (rpm)	Gear 2 (rpm)	Gear 3 (rpm)	Gear 4 (rpm)
1. Gears locked	200	200	200	200
2. Carrier fixed	0	− 100	200	50
3. Total	200	100	400	250

When gear 2 rotates − 100 rpm relative to the carrier, gears 3 and 4 will also rotate. The angular velocity of gear 3 will be

$$\omega_3 = -\frac{N_2}{N_3}\omega_2 = -\frac{40}{20}(-100) = +200$$

and

$$\omega_4 = -\frac{N_2}{N_4}\omega_2 = -\frac{40}{80}(-100) = +50$$

These values are entered into Table 12.1. The results are obtained by adding the results from the first two steps. From this, it is apparent that gear 3 rotates 400 rpm and gear 4 rotates 250 rpm, both in the CCW (+) direction.

EXAMPLE 12.9 *Solution to Example 12.7 Using the Tabulation Method*

Analyze the planetary gear train in Example 12.7 using the tabulation method.

Solution

The problem is to determine the velocity ratio for the gear train, where the velocity ratio is defined by ω_5/ω_6. If we set $\omega_6 = 1$ and solve for the velocity of ω_2, the velocity ratio will be given directly by the value for ω_5. From the problem statement, we also know that $\omega_7 = 0$.

In the table, we will include only gears that rotate about axes that have bearings fixed to the frame. The planet gears rotate about the arm axis, which is skewed relative to the axis of the other gears. Therefore, the angular velocity of the planets is not obtained by a simple algebraic addition of the values from steps 1 and 2.

In the tabulation procedure, we assume that the velocity of the carrier is known. However, in this problem, the velocities of gears 6 and 7 are known. Therefore, we must treat the velocity of the carrier as unknown and solve for it. For step 1, assume that the gears are locked to the carrier and the assembly is turned by $+x$ rpm. This is shown in Table 12.2. In step 2, we must rotate gear 7 such that when the results of the first two steps are added, the final velocity for gear 7 will be zero. Clearly, then, gear 7 must be rotated by $-x$ rpm relative to the carrier. The remaining values in the second row of the table are determined by analyzing the gear train relative to the carrier.

TABLE 12.2 Results for Example 12.9

Step	Carrier 3(rpm)	Gear 2(rpm)	Gear 6(rpm)	Gear 7(rpm)
1. Gears locked	x	x	x	x
2. Carrier fixed	0	$x\frac{N_7}{N_2} = x\frac{76}{20}$	$-x\frac{N_7}{N_4}\frac{N_5}{N_6} = -x\frac{76}{56}\frac{24}{35}$	$-x$
3. Total	x	$x\left[1+\frac{76}{20}\right] = x(4.8)$	$x\left[1-\frac{76}{56}\frac{24}{35}\right] = x(0.0694)$	0

Next sum the results from steps 1 and 2. From the problem statement, we know that $\omega_6 = 1$, and in the table, we can see that the velocity of gear 6 is also given by

$$\omega_6 = x(0.0694) = 1$$

Therefore,

$$x = 14.412$$

and

$$\omega_2 = x(4.8) = 14.412(4.8) = 69.2$$

Therefore the velocity ratio for the planetary gear train is

$$\omega_2/\omega_6 = 69.2$$

which is the same as that computed in Example 12.7.

EXAMPLE 12.10 ***Solution to Ferguson's Paradox Using the Tabulation Method***

An interesting application of planetary gearing is called the Ferguson's paradox.[2] The mechanism is shown in Fig. 12.25. Gear 2 is fixed to the frame, and gear 5 is a planet that rotates relative to the carrier, which is member 6. Gears 2, 3, and 4 have tooth numbers 99, 100, and 101, respectively. All of

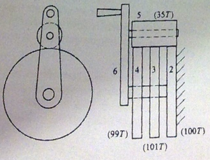


FIGURE 12.25 The planetary gear train for Example 12.10.

the gears are cut from the same blank so that they will all mesh with gear 5, which has 20 teeth. If the arm makes 100 revolutions CCW, determine the number of revolutions made by gears 3, 4, and 5.

Solution Based on the problem statement, we know that the carrier moves 100 revolutions and gear 2 is fixed. As in the previous examples, the first step is to fix all of the gears to the arm and rotate the assembly by 100 turns. This is shown in Table 12.3. In step 2, we must rotate gear 2 such that when the results of the first two steps are added, the final velocity for gear 2 is zero. Therefore, gear 2 must be rotated by − 100 rpm relative to the carrier. The remaining values in the second row of the table are determined by analyzing the gear train relative to the carrier.

Next sum the results from steps 1 and 2. The number of turns made by each gear in the mechanism is shown in the third row of Table 12.3. Notice that gear 3 makes 1 revolution in the direction of the motion of the carrier while gear 4 makes 1 revolution in the opposite direction. Gear 2 is fixed by design. Therefore, as the carrier is turned, gear 3 will rotate very slowly in the direction of the carrier and gear 4 will rotate very slowly in the opposite direction.

TABLE 12.3 Results for Example 12.10

Step	Carrier 6 (revolution)	Gear 2 (revolutions)	Gear 3 (revolutions)	Gear 4 (revolutions)	Gear 5 (revolutions)
1. Gears locked	100	100	100	100	100
2. Carrier fixed	0	− 100	− 100/1.01	− 100/0.99	− 100/35
3. Total	100	0	1	− 1	97.14

REFERENCES

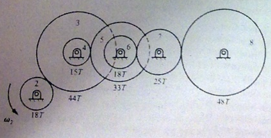
[1]Merritt, H. E., *Gear Engineering*, John Wiley & Sons, New York (1971).

[2]Shigley, J. E., *Kinematic Analysis of Mechanisms*, McGraw–Hill Book Co., New York (1975).

PROBLEMS

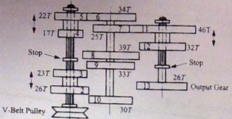
EXERCISE PROBLEMS INVOLVING COMPOUND GEAR TRAINS

12.1 Find the angular velocity of gear 8 if the angular velocity of gear 2 is 800 rpm in the direction shown.

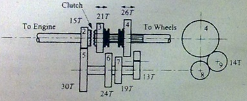


12.2 Find the velocity of gear 8 in Problem 12.1 if the angular velocity of the driver (gear 2) is 300 rpm in the CW direction.

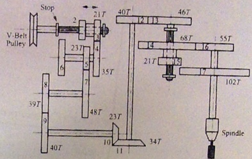
12.3 The gear train given is for a machine tool. Power is input to the gearbox through the pulley indicated, and the output power to the machine table is through gear 13. Gears 2 and 3, 4 and 5, and 11 and 12 are compound gears that can move axially on splined shafts to mesh with various different gears so that various combinations of overall gear ratios (ω_{13}/ω_2) can be produced. Determine the number of ratios possible and the overall gear ratio for each possibility.



12.4 A simple three-speed transmission is shown. The power flow is as follows: (a) First gear: Gear 4 is shifted to mesh with gear 7; power flows through gears 2, 5, 7, and 4. (b) Second gear: Gear 3 is shifted to mesh with gear 6; power flows through gears 2, 5, 6, and 3. (c) Third gear: Gear 3 is shifted so that the clutch teeth on gear 3 mesh with those on gear 2; a direct drive results. (d) Reverse gear: Gear 4 is shifted to mesh with gear 9; power flows through gears 2, 5, 8, 9, and 4. An automobile with this transmission has a differential ratio of 3:1 and a tire outside diameter of 24 in. Determine the engine speed for the car under the following conditions: (i) first gear with the automobile traveling at 15 mph, (ii) third gear with the automobile traveling at 55 mph, and (iii) reverse gear with the automobile traveling at 3.5 mph.



12.5 Part of the gear train for a machine tool is shown. Compound gears 2 and 3 slide on a splined shaft so that gear 3 can mesh with gear 4 or gear 2 can mesh with gear 6. Also, compound gears 14 and 15 slide on a splined shaft so that gear 14 can mesh with gear 16 or gear 15 can mesh with gear 17. (a) If gear 3 meshes with gear 4, what are the two possible spindle speeds for a motor speed of 1800 rpm? (b) Now assume that gear 14 meshes with gear 16, and gear 2 meshes with gear 6. Gears 2, 3, 4, and 6 are standard and have the same diametral pitch. What are the tooth numbers on gears 2 and 6 if the spindle speed is 130 ± 3 rpm?



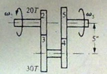
12.6 An internal gear having 160 teeth and rotating CCW at 30 rpm is connected through a gear train to an external gear, which rotates at 120 rpm in the CCW direction. Using the minimum number of gears, select gears from the following list that will satisfy the design requirements. Tooth numbers for the available gears are 20, 22, 25, 30, 32, 34, 35, 40, 50, 55, 60, and 64. There is only one gear with each tooth number, and each gear has the same diametral pitch.

12.7 Re-solve Problem 12.6 if the external gear is concentric with the internal gear (i.e., the rotation axis is the same for both gears) and the external gear rotates clockwise.

12.8 Re-solve Problem 12.6 if the external gear is concentric with the internal gear and the external gear rotates counterclockwise.

12.9 Re-solve Problem 12.6 if the external gear rotates at 50 rpm.

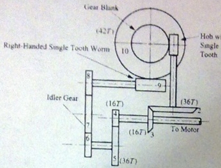
12.10 A gear reducer is to be designed as shown in the figure. Determine the diametral pitch and number of teeth on gears 4 and 5 if the speed of gear 2 (ω_2) is to be 10 times the speed of gear 5 (ω_5). The pitches of the two gears should be as nearly equal as possible, and no gear should have fewer than 15 teeth.



12.11 Re-solve Problem 12.10 if ω_2 is to be 8 times the speed of gear 5 (ω_5).

12.12 Re-solve Problem 12.10 if ω_2 is to be 6.5 times the speed of gear 5 (ω_5).

12.13 The gear train shown is a candidate for the spindle drive of a gear hobbing machine. The gear blank and the worm gear (gear 10) are mounted on the same shaft and rotate together. If the gear blank is to be driven clockwise, determine the hand of the hob. Also determine the velocity ratio (ω_s/ω_h) to cut 72 teeth on the gear blank.



12.14 Assume that the input shaft of a transmission rotates CW at 1800 rpm. The output shaft is driven at 160 rpm in the CCW direction. None of the gears in the transmission is to be an idler, and the gear ratio at any given mesh is not to exceed 3:1. Gears are available that have all tooth numbers between 13 and 85; however, only one gear is available with each tooth number. Select the appropriate gears for the transmission, and sketch the configuration designed. Label the gears and tooth numbers.

12.15 Re-solve Problem 12.14 if the output shaft rotates at 210 rpm in the CCW direction.

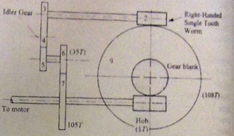
12.16 Re-solve Problem 12.14 if the output shaft rotates at 200 rpm in the CW direction.

12.17 A simple gear reduction is to be used to generate a gear ratio equal to π. Make up a table of possible gear ratios where the maximum number of teeth on either gear is 100. This can be conveniently done using a simple computer program. Identify the gear set that most closely approximates the desired ratio. What is the error?

12.18 A simple gear reduction is to be used to generate the gear ratio 0.467927. Make up a table of possible gear ratios where the maximum number of teeth on either gear is 100. Identify the gear set that most closely approximates the desired ratio. What is the error?

12.19 A simple gear reduction is to be used to generate a gear ratio equal to $\sqrt{2}$. Make up a table of possible gear ratios where the maximum number of teeth on either gear is 100. Identify the gear set that most closely approximates the desired ratio. What is the error?

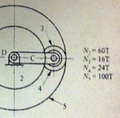
12.20 An alternative gear train is shown as a candidate for the spindle drive of a gear hobbing machine. The gear blank and the worm gear (gear 9) are mounted on the same shaft and rotate together. If the gear blank is to be driven clockwise, determine the hand of the hob. Next determine the velocity ratio (ω_s/ω_h) to cut 75 teeth on the gear blank. Finally, select gears 3 and 5 that will satisfy the ratio. Gears are available that have all of the tooth numbers from 15 to 40.



12.21 A simple gear reduction unit is to be used to generate the gear ratio 2.105399. Make up a table of possible gear ratios where the maximum number of teeth on all gears is 100. Identify the gear set that most closely approximates the desired ratio. Note that this can be done most easily with a computer program. What is the error?

EXERCISE PROBLEMS INVOLVING PLANETARY GEAR TRAINS

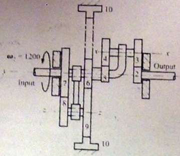
12.22 In the gear train shown, gears 3 and 4 are integral. Gear 3 meshes with gear 2, and gear 4 meshes with gear 5. If gear 2 is fixed and $\omega_5 = 100$ rpm CCW, determine the angular velocity of the carrier.



12.23 Re-solve Problem 12.20 if gear 5 is fixed and $\omega_2 =$ 100 rpm CCW.

12.24 Re-solve Problem 12.20 when $N_2 = 70T$, $N_3 = 35T$, $N_4 =$ 15T, and $N_5 = 12T$.

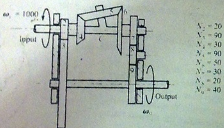
12.25 In the figure given, axis y–y is fixed whereas axes x–x and z–z move with the arm. Gear 7 is fixed to the carrier. Gears 3 and 4, 5 and 6, and 8 and 9 are fixed together, respectively. Gears 3 and 4 move with planetary motion. If the tooth numbers are $N_2 = 16T$, $N_3 = 20T$, $N_4 = 22T$, $N_5 = 14T$, $N_6 = 15T$, $N_7 = 36T$, $N_8 = 20T$, $N_9 = 41T$, and $N_{10} = 97T$, determine the speed and direction of the output shaft.



12.26 Re-solve Problem 12.25 when $N_2 = 16T$, $N_3 = 20T$, $N_4 = 16T$, $N_5 = 20T$, $N_6 = 15T$, $N_7 = 40T$, $N_8 = 15T$, $N_9 = 40T$, and $N_{10} = 95T$.

12.27 Re-solve Problem 12.25 when $N_2 = 14T$, $N_3 = 30T$, $N_4 = 14T$, $N_5 = 30T$, $N_6 = 15T$, $N_7 = 60T$, $N_8 = 15T$, $N_9 = 60T$, and $N_{10} = 135T$.

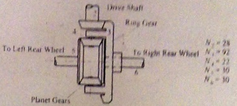
12.28 In the gear train shown, gears 2 and 4, 6 and 7, and 3 and 9 are fixed together. If the angular velocity of the carrier is given, determine the angular velocity of gear 9.



12.29 Re-solve Problem 12.28 if $N_2 = 10T$, $N_3 = 100T$, $N_7 = 20T$, $N_8 = 10T$ and $N_9 = 70T$.

12.30 Re-solve Problem 12.28 but assume that the shaft connecting gears 3 and 9 is the input shaft and the shaft of the carrier is the output shaft. Assume $\omega_9 = 500$ rpm CCW and compute ω_c.

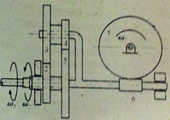
12.31 The differential for a rear-wheel-driven vehicle is shown schematically. If the drive shaft turns at 900 rpm, what is the speed of the vehicle if neither wheel slips and the outside diameter of the wheels is 24 in?



12.32 Assume that the vehicle in Problem 12.31 is stopped so that the right wheel sits on a small icy patch and can spin freely while the left wheel does not spin. Determine the angular velocity of the right wheel if the angular speed of the drive shaft is 500 rpm.

12.33 Assume that the vehicle in Problem 12.31 is traveling at 35 mph and turns around a curve with a radius of 50 ft from the centerline of the vehicle. The center-to-center distance between the treads of the right and left wheels is 60 in. Compute the rotational speed of each rear wheel, the rotational speed of the ring gear, and the rotational speed of the drive shaft.

12.34 In the mechanism shown, derive an expression for the angular velocity of gear 7 (ω_7) in terms of ω_2 and ω_5 and the tooth numbers N_2, N_3, N_4, N_5, N_6, and N_7. Take counterclockwise viewing from the left as positive for the rotation of gears 2, 3, 4, 5, and 6. Viewed from the front of the page, take counterclockwise as the positive direction for gear 7.



12.35 In Problem 12.34, assume that $\omega_2 = 100$ rpm, $\omega_5 = 60$ rpm, $N_2 = 40T$, $N_3 = 60T$, $N_4 = 30T$, $N_5 = 70T$, $N_6 = 8T$, and $N_7 = 50T$. Determine the angular velocity of both gears 6 and 7.

12.36[1] The figure on the next page shows a schematic diagram of a semiautomatic transmission from the Model-T automobile. This was the forerunner of today's automatic transmission. A plate clutch, two banded clutches, and a system of pedals and levers (used to engage and disengage these plate and band clutches) operated in the proper sequence are shown in the table. Determine the output/input speed ratio for each condition.

[1]Problem courtesy of Dr. Michael Stanisic, Notre Dame University.