whatever market price may prevail. In this circumstance the factor supply function is a vertical straight line with abscissa equal to the aggregate factor [illegible]. A more interesting case is one in which consumers gain utility from [illegible] some of or all their factor stocks.

[illegible] it was assumed in Sec. 2-4 that utility is a function of leisure and [illegible]

$$[illegible] = [illegible] - W, [illegible]$$

where T is the total amount of available time (the length of the period for which the utility function is defined) and W the amount of work performed in terms of hours. It was shown that the utility-maximizing individual allocates his time between work and leisure in such fashion that

$$\frac{g_{[illegible]}}{g_{[illegible]}} = r \qquad (6\text{-}13)$$

where r is the wage rate and g_i is the partial derivative of the utility function with respect to its ith argument. The g_i's depend upon income and the amount of work performed. Since $y = rW$, (6-13) contains only the variables r and W. Solving (6-13) for W and setting $W = S_i$, the labor supply function of the ith individual is

$$S_i = S_i(r)$$

The supply function states the amounts of work that the individual is willing to perform as a function of the wage rate. The aggregate supply function is obtained by summing the individual supply functions. If there are n individuals who are willing to supply labor at some wage rate, the aggregate supply function is

$$S = \sum_{i=1}^{n} S_i(r) = S(r)$$

The supply curve may have negative slope, positive slope, or both. If individuals value leisure highly and are more concerned with increasing their time for leisure than raising their incomes, the supply curve of labor may be negatively sloped: the higher the wage, the less work is performed.

Market Equilibrium

Given the demand and supply functions for an input the equilibrium price-quantity combination is determined by invoking the equilibrium condition $D = S$. Market forces similar to those discussed in Sec. 6-4 will change the existing situation whenever the actual price differs from the equilibrium price. Equilibrium is reached only when the quantity demanded equals the quantity supplied. As in product markets, no participant can improve his position by recontracting after equilibrium has been reached.

Since the equilibrium price-quantity combination must lie on both the demand and supply curves, it must also satisfy the producer's equilibrium conditions from which the demand curve is derived. The equilibrium price of an input is always equal to the value of its marginal product; i.e., the value of the marginal dollar spent on inputs is the same in every use.[1] This equality is a necessary condition for profit maximization, and every entrepreneur can reach his optimum point in a perfectly competitive market if his second-order conditions for maximization are fulfilled.

6-7 THE EXISTENCE AND UNIQUENESS OF [illegible]

Thus far, the analysis of market equilibrium has been based [illegible] the assumption that a unique price-quantity equilibrium exists for each [illegible] market under investigation. It is not difficult to construct examples [illegible] this existence assumption is violated: supply and demand are not equal at any nonnegative price-quantity combination. Likewise, examples exist in which the assumption of uniqueness is violated: supply and demand are equal at more than one nonnegative price-quantity combination. This section is limited to general observations and the discussion of some specific cases. The problems of existence and uniqueness are considered more [illegible] within a multimarket framework in Chap. 10.

Existence

A competitive market equilibrium will exist if there is one or more nonnegative prices at which demand and supply are equal and nonnegative. In terms of the conventional diagram, equilibrium will exist if the demand and supply curves have at least one point in common in the nonnegative quadrant.

Three situations in which the supply and demand curves have no point in common are pictured in Fig. 6-7. Supply exceeds demand at every nonnegative price for the case pictured in Fig. 6-7*a*. No equilibrium exists according to the definition given above. The definition of equilibrium is easily broadened to cover this case. Let $p = 0$ if $S(0) > D(0)$. A *free good* has a price of zero and is characterized by an excess of supply over demand. Consumers can get all they want for nothing. Air and water may be considered free goods. Up to some critical point water may be there for the taking. Beyond this point purification and transportation may become necessary and lead to a positive supply price.

Fig. 6-7*b* covers a case in which the demand price is less than the supply price at each nonnegative output. The amounts that consumers are willing to

[1] This has an analog in the theory of consumer behavior. Recall that $f_1 - \lambda p_1 = 0$ is one of the equilibrium conditions for the consumer, where f_1 is the marginal utility of the first good and λ is the marginal utility of money. Then $f_1(1/\lambda) = p_1$, or the price of the commodity must equal its marginal utility multiplied by the additional amount of money that has to be paid per unit of additional utility $(1/\lambda)$.

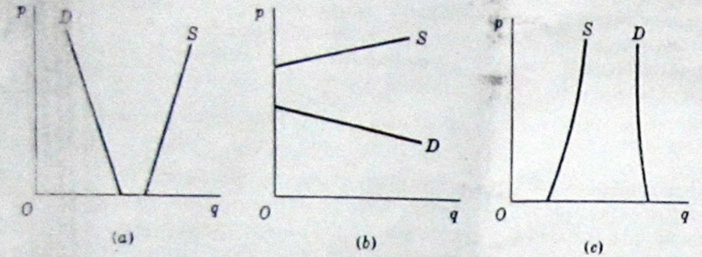

Figure 6-7

pay are inadequate to compensate producers. Market equilibrium does not exist by the definitions thus far given. Again it is possible to broaden the definitions to cover such cases. An equilibrium exists with zero output if supply price exceeds demand price for all nonnegative outputs. It is technologically possible to produce solid-gold school-lunch boxes, but none are produced because parents are not willing to pay enough to allow producers to cover their costs.

The free-good and zero-production cases are meaningful. They are covered by the general methods described in Chap. 10. Many other cases in which equilibrium cannot be achieved are the result of poor model specification. If such cases are encountered, the assumptions of their underlying producer and consumer models must be altered in order to provide a meaningful framework for analysis. Figure 6-7c provides an example. Demand exceeds supply for every price, and there is no meaningful interpretation that can be placed upon this situation.

Uniqueness

It is possible that more than one equilibrium exists, i.e., that demand and supply are equal at more than one nonnegative price-quantity combination. Points A and B in Fig. 6-8a are both equilibria. The demand curve is downward sloping in the normal fashion, but the supply curve bends back as price increases. Quantity is a single-valued function of price, but price is not a single-valued function of quantity. Some economists have found evidence that a "backward-bending" supply curve exists for labor markets in some developing countries. The supply curve is positively sloped at relatively low wage rates, and an increase in the wage rate brings forth an increased supply of labor. However, as the wage rate continues to increase and the income of each worker increases, a point is reached at which the workers prefer leisure to yet more income.

Let δ be the difference in the slopes of the demand and supply curves: $\delta = D'(p) - S'(p)$. If the demand curve is negatively sloped throughout and

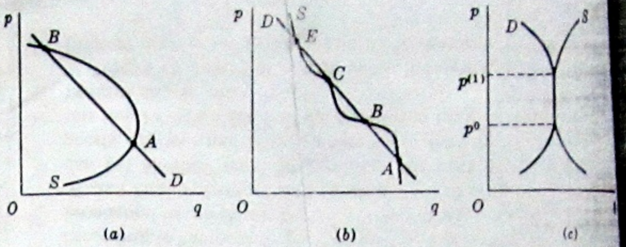

Figure 6-8

the supply curve is positively sloped throughout, $\delta < 0$ for all prices and there cannot be more than one equilibrium point. If $\delta < 0$ at an equilibrium price, demand will be less than supply at a price slightly higher than p^0 and will be greater than supply at a price slightly lower than p^0. As long as $\delta < 0$ the demand curve will remain to the left of the supply curve at prices above p^0 and to the right at prices below p^0. Hence, there cannot be a second equilibrium point. A similar argument can be utilized to prove that there cannot be more than one equilibrium point if $\delta > 0$ throughout.

In Fig. 6-8a $\delta < 0$ for equilibrium point A. At B the demand and supply curves are both negatively sloped. The demand curve is more steeply sloped than the supply curve and $\delta > 0$ at B.† Four equilibrium points are shown in Fig. 6-8b. The supply curve has negative slope throughout, reflecting external economies. The values of δ are negative at equilibrium point A, positive at B, zero at C, and negative at E. In general, ignoring equilibrium points at which $\delta = 0$, δ must alternate in sign at adjacent equilibrium points. Equilibrium points with $\delta = 0$ may lie between or on either side of the points with alternating sign.

There will be a range of equilibrium points with $\delta = 0$ if the demand and supply curves are coincident for all or a portion of their lengths. Such a case is shown in Fig. 6-8c. Here, the equilibrium quantity is unique, but any price from p^0 through $p^{(1)}$ is an equilibrium price.

6-8 THE STABILITY OF EQUILIBRIUM

Equilibrium price and quantity are determined by the equality of demand and supply. Equilibrium is characterized by the acquiescence of buyers and sellers in the status quo: no participant in the market has an incentive to modify his behavior. However, the existence of an equilibrium point does not guarantee

† The derivatives $D'(p)$ and $S'(p)$ are functions of price. Hence, a steeper downward slope of the demand curve means that $D'(p)$ [illegible]

that it will be attained. It ... established if the market ... There is also no reason ... equilibrium price. Moreover ... shift the demand curve ... factors tend to disturb ... defines a new equilibrium ... attained.

In general, a disturbance ... different from the equilibrium ... results in a return to equilibrium ... assumed in the discussion ... brium was stable.

Static Stability

A disturbance usually ... example, if the actual ... may consist of some ... analysis abstracts from ... only the nature of ... equilibrium.

Define

as the excess demand ... price p^0, negative ... assumptions about the ... stability condition is ... bids if excess demand ... negative. If this be ... rise diminishes excess ...

† This is not a rigor... See P. A. Samuelson ... versity Press, 1948, pp ...

‡ Rewrite the den... define excess demand ... states that reduces ... equilibrium is stable ... demand curve has ... according to both ... equilibrium will be st...

that it will be attained. There is no guarantee that the equilibrium price will be established if the market is not in equilibrium when the contracting begins. There is also no reason to assume that the initial price will happen to be the equilibrium price. Moreover, changes in consumer preferences will generally shift the demand curve, and innovations will shift the supply curve. Both factors tend to disturb an established equilibrium situation. The change defines a new equilibrium, but there is again no guarantee that it will be attained.

In general, a disturbance denotes a situation in which the actual price is different from the equilibrium price. An equilibrium is *stable* if a disturbance results in a return to equilibrium and *unstable* if it does not.[1] It was implicitly assumed in the discussion of equilibrium in Sec. 6-4 that the market equilibrium was stable.

Static Stability

A disturbance usually creates an adjustment process in the market. For example, if the actual price is less than the equilibrium price, the adjustment may consist of some buyers raising their bids for the commodity. Static analysis abstracts from the time path of the adjustment process and considers only the nature of the change, i.e., whether it is toward, or away from, equilibrium.

Define

$$E(p) = D(p) - S(p)$$

as the excess demand at price p. In Fig 6-9 excess demand is positive at the price p^0, negative at the price $p^{(1)}$. Stability conditions are derived from assumptions about the market behavior of buyers and sellers. The *Walrasian stability condition* is based on the assumption that buyers tend to raise their bids if excess demand is positive and sellers tend to lower their prices if it is negative. If this behavior assumption is correct, a market is stable if a price rise diminishes excess demand,[2] i.e., if

$$\frac{dE(p)}{dp} = E'(p) = D'(p) - S'(p) < 0 \qquad (6\text{-}14)$$

[1] This is not a rigorous definition of stability and is only one of several alternative definitions. See P. A. Samuelson, *Foundations of Economic Analysis* (Cambridge, Mass.: Harvard University Press, 1948, pp. 260–262.

[2] Rewrite the demand and supply functions in inverse form $p_d = D^{-1}(q)$, $p_s = S^{-1}(q)$, and define excess demand price $F(q) = p_d - p_s = D^{-1}(q) - S^{-1}(q)$. The *Marshallian stability condition* states that producers will raise their output when $F(q) > 0$ and lower it when $F(q) < 0$. Thus equilibrium is stable in the Marshallian sense if $dF(q)/dq = F'(q) = D^{-1\prime}(q) - S^{-1\prime}(q) < 0$. If the demand curve has negative slope and the supply curve positive slope, an equilibrium is stable according to both definitions. If the demand and supply curves have slopes of the same sign, an equilibrium will be stable according to one definition and unstable according to the other.

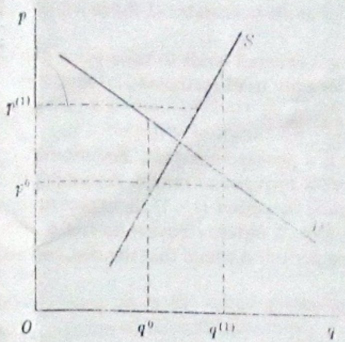

Figure 6-9

This condition is satisfied automatically if the demand curve has negative slope and the supply curve has positive slope. If both are positively sloped, the supply curve must be flatter than the demand curve $[S^{-1\prime}(q) < D^{-1\prime}(q)]$ to satisfy (6-14). If both are negatively sloped, the supply curve must be steeper than the demand curve.

The negatively sloped supply curve depicted in Fig. 6-8b yields four equilibrium points. The successive equilibrium points A, B, and E are alternately stable and unstable by the Walrasian behavior assumption (6-14). The supply curve is steeper than the demand curve at A, and the equilibrium is stable at this point. Another intersection B can exist only if the supply curve becomes less steep than the demand curve; B is therefore unstable. By similar reasoning, D is again stable. The stability condition (6-14) is not sufficient to cover equilibrium point C. Excess demand is positive at prices less than p_e and also at prices higher than p_e. The price will tend to rise for downward or upward deviations from equilibrium. Point C is classed as *semistable*.

Dynamic Stability: Lagged Adjustment

The static stability condition (6-14) is stated in terms of the rate of change of excess demand with respect to price. Nothing is said about the time path of adjustment. One might not expect instantaneous adjustments in the present model. If the initial price is not equal to the equilibrium price, it changes, and recontracting takes place. If the new price is still different from the equilibrium price, it is again forced to change. The dynamic nature of the recontracting may be formalized in a model in which recontracting takes place during periods of fixed length, say, one hour, with the auctioneer announcing the new price at the beginning of each period. The analysis of dynamic stability investigates the course of price over time, i.e., from period to period.[1] Equilibrium is stable in the dynamic sense if the price converges to (or approaches) the equilibrium price over time; it is unstable if the price change is away from equilibrium.

The assumption that a positive excess demand tends to raise price can be modeled in many different ways. A commonly used mathematical model is

$$p_t - p_{t-1} = kE(p_{t-1}) \tag{6-15}$$

where p_t is the price in period t and k is a positive constant. Equation (6-15) expresses one possible type of behavior for buyers and sellers. Assuming that there is a positive excess demand $E(p_{t-1})$ in period $(t-1)$, it expresses the assumption that an excess demand of $E(p_{t-1})$ induces buyers to bid a price $p_t = p_{t-1} + kE(p_{t-1}) > p_{t-1}$ in the following period. Assume that the demand and supply functions are

$$D_t = ap_t + b \tag{6-16}$$

$$S_t = Ap_t + B \tag{6-17}$$

Excess demand in period $(t-1)$ is

$$E(p_{t-1}) = (a - A)p_{t-1} + b - B$$

Substituting this into (6-15),

$$p_t - p_{t-1} = k[(a - A)p_{t-1} + b - B]$$

and

$$p_t = [1 + k(a - A)]p_{t-1} + k(b - B) \tag{6-18}$$

The first-order difference equation (6-18) describes the time path of price on the basis of the behavior assumption contained in (6-15). Given the initial condition $p = p_0$ when $t = 0$, its solution is

$$p_t = (p_0 - p_e)[1 + k(a - A)]^t + p_e \tag{6-19}$$

where

$$p_e = \frac{b - B}{A - a}$$

is the equilibrium price determined from (6-16) and (6-17) by setting $D_t - S_t = 0$ and solving for $p_t = p_e$. The equilibrium is stable if the actual price level approaches the equilibrium level as t increases. The price level converges to p_e without oscillations if $0 < 1 + k(a - A) < 1$. The right-hand side of this inequality holds if

$$a < A \tag{6-20}$$

The left-hand side holds if

$$k < \frac{1}{A - a}$$

[1] The prices which are recorded from period to period are potential, rather than realized, until equilibrium is reached. As long as $D \neq S$, none of the contracts is executed, and recontracting continues.

Condition (6-20) is automatically fulfilled if the supply curve has positive slope ($A > 0$). The price level moves upward over time if the initial price is less than the equilibrium price and downward if it is greater. If the slope of the supply curve is negative, stability requires that the slope of the demand curve ($1/a$) be algebraically greater than the slope of the supply curve ($1/A$); i.e., the supply curve must cut the demand curve from above. Equilibrium is unstable if the supply curve cuts the demand curve from below, and any deviation from equilibrium is followed by increasing deviations from it. If k is sufficiently large and $a - A$ is negative, $1 + k(a - A)$ is also negative, and the price level must oscillate over time.[1]

Both static and dynamic stability depend upon the slopes of the demand and supply curves. Dynamic stability depends in addition on the magnitude of the parameter k which indicates the extent to which the market adjusts to a discrepancy between the quantities demanded and supplied per unit of time. A large k indicates that buyers and sellers tend to "overadjust": if excess demand is positive, bidding by buyers is sufficiently active to raise the price above the equilibrium level. Each adjustment is in the right direction, but is exaggerated in magnitude. Dynamic analysis thus takes into account the strength of reactions to disturbances.

The dynamic stability of equilibrium can be analyzed diagrammatically in the following fashion. Plotting price along the horizontal axis, the dotted line in Fig. 6-10a represents the excess demand function. Assuming that $k < 1$, the solid line represents $kE(p_{t-1})$. The 45-degree line in Fig. 6-10b represents the locus of points defined by $p_t = p_{t-1}$. The function

$$p_t = p_{t-1} + kE(p_{t-1}) = f(p_{t-1})$$

is obtained by adding the ordinates (corresponding to the same abscissa) of

[1] If $1 + k(a - A)$ is greater than -1 (but less than zero), the amplitude of the oscillations decreases over time, and the time path approaches the equilibrium level. If it is less than -1, the market is subject to increasing price fluctuations.

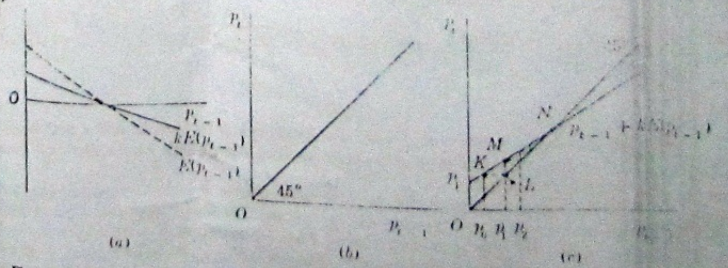

Figure 6-10

the solid lines in Figs. 6-10a and 6-10b. The result is shown in Fig. 6-10c. Assume that the initial price is p_0. The price in the following period, p_1, is given by the ordinate of the point on $f(p_{t-1})$ directly above p_0. In order to calculate the price in the following period, p_1 is transferred to the horizontal axis by moving a horizontal line from K to L. L lies on a 45-degree line, and the abscissa of each point on it equals its ordinate. The price p_2 is found by moving vertically to M on $f(p_{t-1})$. All subsequent prices are found in this manner. The price level converges in the present example to the equilibrium price given by the intersection of $f(p_{t-1})$ and the 45-degree line.[1] The stability of equilibrium depends upon the slope of the excess demand function and the magnitude of k. If the excess demand function in Fig. 6-10a were positively sloped, the function $f(p_{t-1})$ would cut the 45-degree line from below, and the equilibrium would be unstable. If the excess demand function had negative slope, as in Fig. 6-10a, but k were very large, $f(p_{t-1})$ would have negative slope, and the price level would oscillate.

The static and dynamic approaches to stability are fundamentally different. Static stability need not imply dynamic stability, but dynamic stability implies static stability. The reason for this discrepancy is that dynamic analysis is a more inclusive tool for investigating the properties of equilibrium. Static analysis concerns itself only with the direction of the adjustment and neglects the magnitude of the adjustment from period to period.

Let

$$D_t = -0.5p_t + 100$$

$$S_t = -0.1p_t + 50$$

and let $k = 6$. The equilibrium is stable in the static Walrasian sense if $D'(p) - S'(p) < 0$. Substituting from the demand and supply functions, $-0.5 - (-0.1) = -0.4 < 0$. Dynamic stability requires $-1 < 1 + k(a - A) < 1$. Substituting the appropriate values gives

$$1 + k(a - A) = -1.4$$

and the required left-hand inequality does not hold. The market will exhibit explosive oscillations.

Dynamic Stability: Continuous Adjustment

Equation (6-15) describes a price adjustment process that occurs over discrete intervals of time. An alternative approach is based on the assumption that

[1] It can be easily verified that point N is the equilibrium point. At N, $p_t = p_{t-1}$ (for the 45-degree line) and $p_t = p_{t-1} + kE(p_{t-1})$. Substituting p_{t-1} for p_t,

$$p_{t-1} = p_{t-1} + kE(p_{t-1})$$

or $kE(p_{t-1}) = 0$. Excess demand equals zero at point N.

adjustment takes place continuously. Equation (6-15) is then replaced by

$$\frac{dp}{dt} = kE(p) \tag{6-21}$$

where k and $E(p)$ have the same meaning as before.[1] Substituting the demand and supply functions (6-16) and (6-17), (6-21) becomes

$$\frac{dp}{dt} = k(a - A)p + k(b - B) \tag{6-22}$$

which is a first-order differential equation. Its solution (see Sec. ...) is

$$p = (p_0 - p_e)e^{k(a-A)t} + p_e$$

where p_0 is the initial price at $t = 0$ and $e = 2.71828\ldots$ is the base of the system of natural logarithms.

The equilibrium price p_e is dynamically stable, that is, $p \to p_e$ as $t \to \infty$, if $(a - A) < 0$, which will be the case if the demand function is negatively and the supply function positively sloped. The magnitude of the adjustment coefficient influences the speed with which convergence or divergence takes place, but in contrast to the lagged adjustment model it plays no role in determining whether an equilibrium is stable or not. The static and dynamic stability conditions are identical in this case.

An equilibrium point is *locally* stable if the system returns to it, given a small initial deviation from equilibrium. It is *globally* stable if the system returns to it for any initial deviation from equilibrium. Linear models such as (6-22) have unique equilibrium points in general, and if they are locally stable, they are also globally stable. Nonlinear models may have several equilibrium points, and, in any event, the local stability of an equilibrium point does not guarantee its global stability.

A linear approximation is useful in determining the local stability of nonlinear models. Assume that the excess demand function $E(p)$ is some complicated function of p so that the differential equation (6-21) is difficult or impossible to solve directly. The approximate equality

$$\frac{E(p) - E(p_e)}{p - p_e} \approx E'(p_e) \tag{6-23}$$

where p_e is an equilibrium price, follows from the definition of a derivative. In the limit, as $p \to p_e$, (6-23) holds exactly, and for small deviations of p from p_e the approximation may be expected to be good. Substituting $E(p_e) = 0$, solving (6-23) for $E(p)$, and substituting the result on the right-hand side of (6-21),

$$\frac{dp}{dt} = kE'(p_e)(p - p_e)$$

[1] The value of p is defined for all values of t. It is customary in this case to omit the subscript t. The dependence of p on t may be indicated explicitly by writing $p(t)$.

which is a linear equation since $E'(p_e)$, the derivative of excess demand evaluated at p_e, is a constant. The root of the characteristic equation (valid in the neighborhood of p_e) is $kE'(p_e)$. Thus, if the excess demand function is negatively sloped in the neighborhood of p_e, the equilibrium is locally stable. The static and dynamic conditions again are identical.

The existence of global stability can often be ascertained by a technique known as *Liapunov's direct method*. First find a Liapunov function, $V(p)$, such that $V(p) > 0$ if $p \neq p_e$ and $V(p_e) = 0$. If dV/dt is negative whenever $p \neq p_e$, the equilibrium solution is globally stable.[1] An appropriate Liapunov function is often provided by

$$V(p) = (p - p_e)^2$$

the squared distance of the actual point p at time t from the equilibrium point.

For illustration, consider the nonlinear excess demand function $E = b/p - a$ where $p_e = b/a$ with $a, b > 0$, and

$$\frac{dp}{dt} = k\left(\frac{b}{p} - a\right)$$

Differentiating $V(p)$,

$$\frac{dV}{dt} = 2(p - p_e)\frac{dp}{dt}$$

Substituting for p_e and dp/dt,

$$\frac{dV}{dt} = -\frac{2k(ap - b)^2}{ap}$$

which is negative for all $p \neq p_e$ since k, a, and p are positive. Thus, an equilibrium for this model is globally stable.

6-9 DYNAMIC EQUILIBRIUM WITH LAGGED ADJUSTMENT

Producers' supply functions show how they adjust their outputs to the prevailing price. Since production takes time, the adjustment may not be instantaneous, but may become perceptible in the market only after a period of time. Agricultural commodities often provide good examples of lagged supply. Production plans are made after the harvest. The output corresponding to these production plans appears on the market a year later. Assume that the demand and supply functions are

$$D_t = ap_t + b \tag{6-24}$$

$$S_t = Ap_{t-1} + B \tag{6-25}$$

[1] More advanced treatises distinguish between *stability* and *asymptotic stability*. See J. La Salle and S. Lefschetz, *Stability by Liapunov's Direct Method* (New York: Academic, 1961), pp. [illegible].

The market is in dynamic equilibrium if the price remains unchanged from period to period, i.e., if $p_t = p_{t-1}$. Equating (6-24) and (6-25) yields the unique equilibrium price $p_e = (B - b)/(a - A)$. The quantity demanded in any period depends upon the price in that period, but the quantity supplied depends upon the price in the previous period. It is assumed that the quantity supplied in period t is always equal to the quantity demanded in that period; that is, p_t adjusts to bring about the equality of D_t and S_t as soon as S_t appears on the market. This implies that no producer is left with unsold stocks and no consumer with an unsatisfied demand. Therefore

$$D_t - S_t = 0$$

Substituting from (6-24) and (6-25),

$$ap_t + b - Ap_{t-1} - B = 0$$

Solving for p_t,

$$p_t = \frac{A}{a}p_{t-1} + \frac{B - b}{a} \tag{6-26}$$

Assuming that the initial condition is given by $p = p_0$ when $t = 0$, the solution of the first-order difference equation (6-26) is

$$p_t = (p_0 - p_e)\left(\frac{A}{a}\right)^t + p_e \tag{6-27}$$

The solution (6-27) describes the path of the price as a function of time. Some of the possible time paths are illustrated in Figs. 6-11*a* and 6-11*b*.

Assume that the initial supply does not equal the equilibrium amount as a result of a disturbance such as a drought. Let the initial supply equal q_0 in Fig.

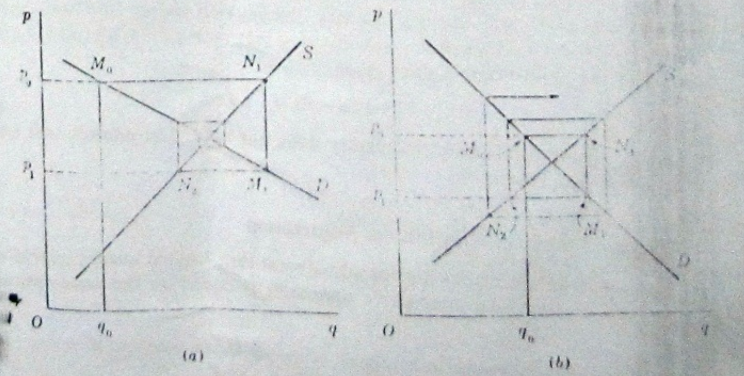

Figure 6-11

6-11a. The corresponding initial price is p_0. Consumers demand p_0M_0, and this quantity equals the initial supply. The price p_0 induces entrepreneurs to supply the quantity p_0N_1 in the next period. The price falls instantaneously to p_1. The quantity demanded is then p_1M_1 (which equals p_0N_1, the quantity supplied in that period). In the following period the price p_1 induces a supply of p_1N_2. This process continues indefinitely, producing a cobweb pattern. The price level fluctuates, but converges to the equilibrium level indicated by the intersection of the demand and supply curves. The same mechanism operates in Fig. 6-11b, but the price fluctuations tend to become larger and larger: the market is subject to explosive oscillations.

The market is dynamically stable if $p_t \rightarrow p_e$ as $t \rightarrow \infty$. If the absolute value of the quotient (A/a) is less than one, the first term on the right of (6-27) will vanish as $t \rightarrow \infty$, and the market will be dynamically stable. If the slopes of the demand $(1/a)$ and supply curves $(1/A)$ have opposite sign, price will oscillate about the equilibrium price level. If the slope of the demand curve has smaller absolute value than the slope of the supply curve, $1/|a| < 1/|A|$, the oscillations will decrease in amplitude, and the market is dynamically stable as shown in Fig. 6-11a. If the slope of the demand curve has greater absolute value than the slope of the supply curve, $1/|a| > 1/|A|$, the oscillations will increase in amplitude, and the market is dynamically unstable as shown in Fig. 6-11b. Finally, if the slopes of the demand and supply curves are equal in absolute value, $1/|a| = 1/|A|$, the oscillations will have constant amplitude, and the market is dynamically unstable.

If the demand and supply curves slope in the same direction, A/a is positive, and the price level will not oscillate, but will either increase or decrease continually.[1] The same conditions hold as above: price will converge

[1] The price may remain constant if the demand and supply curves coincide. No unique equilibrium is defined in this case. See Sec. 6-7.

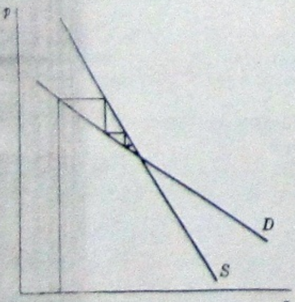

Figure 6-12

to its equilibrium value if the demand curve has smaller absolute slope than the supply curve (Fig. 6-12), and will diverge in either an upward or downward direction if the demand curve has greater absolute slope.

The conditions for dynamic stability are not the same as in the simple dynamic case. Buyers and sellers react to excess demand in the simple dynamic case. Excess demand is zero in cobweb situations. Buyers react to given supplies in terms of the prices they offer. Sellers respond to given supplies in terms of the prices they offer. Sellers respond to given prices in terms of the quantities they supply in the following period.

6-10 A FUTURES MARKET

Futures markets have been instituted for some commodities with uncertain future prices. Buyers and sellers agree to transact at a specified price at a future date. Thereby, the future price for such transactions is known with certainty. Futures markets are common for agricultural commodities. A risk-averse farmer who sells for future delivery can avoid price uncertainty. A processor of agricultural products who buys for future delivery can contract for sale of his output, given a certain input cost. People who buy and sell for these reasons are said to *hedge* against price uncertainty. Others who have no direct interest in the commodity may also buy and sell in a futures market. A buyer (seller) can sell (buy) at the actual market price at the future date in order to cover his contract. Such a person will participate in a futures market if he can increase his expected utility by buying or selling the lottery offered by the market.

Differing expectations as to future price can lead to futures-market transactions. It is assumed here that expectations are identical in the sense that everyone expects that future price will be one of the n values $(p_1, \ldots, p_n)$ with the respective probabilities $(v_1, \ldots, v_n)$. In order to emphasize that a futures market does not require some participants who prefer risk, an example is given for which all buyers and sellers are risk averse, though not to the same degree, and all obey the von Neumann–Morgenstern axioms (see Sec. 3-8).

Hedging

Consider a farmer who produces the commodity under investigation. Let his cost function $C(q)$ be strictly convex, and his utility function $U(\pi)$ be strictly concave. If the farmer sells in the futures market at the going price p^*, he maximizes utility by equating this price to his MC. If he does not sell in the futures market, his first-order condition for expected utility maximization [see (5-21)] is

$$\frac{dE[U(\pi)]}{dq} = \sum_{i=1}^{n} v_i U'(\pi_i)[p_i - C'(q)] = 0 \qquad (6\text{-}28)$$

Let U_0 be the maximum utility value determined from (6-28). The utility level from futures-market participation is

$$U^* = U[p^*q^* - C(q^*)] = V(p^*)$$

where q^* is a solution for $p^* = C'(q^*)$. Clearly, $dU^*/dp^* > 0$. Let p_0^* be the solution of $U_0 = V(p^*)$. For $p^* < p_0^*$ the farmer will not sell in the futures market; he prefers price uncertainty to the certainty provided by the futures market. For $p^* > p_0^*$ he will sell his entire output as determined by his MC function; he then prefers the certainty of the futures market.

For illustration let $U = \ln(\pi + 10)$ and $C = 0.5q^2$ with $p_1 = 4$, $p_2 = 8$, and $r_1 = r_2 = 0.5$. An approximate solution for (6-28) is $q_0 \approx 5.246$ and $U^0 \approx 3.245$. Also,

$$V(p_0^*) = \ln(0.5p_0^{*2} + 10) = 3.245$$

has the solution $p_0^* \approx 5.598$. The farmer's futures-market supply function[1] is

$$S = 0 \qquad \text{if} \qquad p^* < 5.598$$
$$S = p^* \qquad \text{if} \qquad p^* > 5.598$$

The construction of a futures-market demand function for a processor of agricultural output is left for the reader (see Exercise 6-13).

Risk Assumption

A person without a direct interest in a commodity may buy or sell in its futures market if he can increase his utility. He avoids taking delivery of or having to provide the commodity through an offsetting transaction at the prescribed future date. Let his utility be a function of his asset position $U = U(A)$ with an initial position of $U_0 = U(A_0)$. Let D denote his excess demand in the futures market such that $D > 0$ means that he is buying for future delivery at p^*, and $D < 0$ means that he is selling. His expected utility is

$$E[U(A)] = \sum_{i=1}^{n} r_i U[A_0 + (p_i - p^*)D] \tag{6-29}$$

The first-order condition for his expected utility maximization is [illegible]

$$\frac{dE[U(A)]}{dD} = \sum_{i=1}^{n} r_i U'(A_0 + (p_i - p^*)D)(p_i - p^*) = 0 \tag{6-30}$$

The participant's excess demand function is obtained by solving (6-30) for $D = D(p^*)$. Let p_0^* be a solution for $D(p^*) = 0$. If $p^* > p_0^*$, the participant will sell in the futures market, and if $p^* < p_0^*$, he will buy.

[1] Each farmer's [illegible] supply function has a discontinuity since output jumps from 5.246 to

For example, let $U(A) = \ln(A)$. Substituting into (6-30) gives

$$D = \frac{6 - p^*}{(8 - p^*)(p^* - 4)} A_0$$

For any given $4 < p^* < 8$, futures-market purchases will equal a fixed proportion of the participant's assets. Assume that this proportion cannot exceed 1, which occurs at $p \approx 4.44$. Assume that there are 10,000 identical participants each with $A_0 = 9056.25$ who serve as buyers in the futures market, and 1000 identical farmers as described above who serve as sellers. Equating aggregate demand to aggregate supply

$$\frac{(6 - p^*)}{(8 - p^*)(p^* - 4)} 90{,}562.5 = 1000p^*$$

which has the solution $p^* = 5.75$ and $q = 5750$.

6-11 SUMMARY

The theory of perfect competition analyzes the factors that determine price and quantity in markets in which (1) the product is homogeneous and buyers are uniform, (2) buyers and sellers are numerous, (3) buyers and sellers possess perfect information, (4) there are free entry and exit for both buyers and sellers in the long run. The participants in the market act as if they had no influence on the price, and each individual regards it as a given parameter.

The price and the quantity bought and sold are determined by supply and demand. The aggregate demand function is derived from the demand functions of individual consumers, which, in turn, are derived from the individual consumers' first-order conditions for utility maximization. The aggregate supply function is derived from individual supply functions which are based on the individual firms' first-order conditions for profit maximization. Equilibrium is attained when demand equals supply. The equality of demand and supply guarantees that buyers' and sellers' desires are consistent. The analysis of a perfectly competitive market is extended to specific and ad valorem sales taxes.

The analysis of perfectly competitive factor markets is similar to the analysis of commodity markets. The equilibrium price-quantity combination is determined by demand and supply, and the equality of demand and supply ensures the consistency of buyers' and sellers' desires. The demand function for a factor is derived from the individual firms' first-order conditions for profit maximization. The supply function for a primary input such as labor is derived from the individual laborers' first-order conditions for utility maximization. Equilibrium in a factor market ensures that the price of a factor equals the value of its marginal product.

The existence of demand and supply functions does not necessarily imply that demand and supply are equal at [illegible] nonnegative price-quantity