INDIAN INSTITUTE OF TECHNOLOGY, BHUBANESWAR

ASSIGNMENT: 01

Course: ME 20001 Dynamics (Second Year Autumn Semester 2011)

|Kinematics of Rigid Body: Translation, Rotation, General Plane Motion, Absolute and Relative Motion, Velocity & Acceleration|

Translation: It is defined as any motion, in which every line in the rigid body remains parallal to its original position at all times.

(a) Rectilinear translation: All points in the body move in parallel straight lines

(b) Currilinear translation: All points move on congruent curves.

In rectilinear translation, velocities and accelerations of all particles of the rigid body are same and parallel.

For uniform rectilinear motion: $x = x_0 + vt$

For uniform accelerated motion: $x = x_0 + v_0 t + \frac{1}{2} f t^2$

$$v = v_0 + ft$$

$$v^2 = v_0^2 + 2f(x - x_0)$$

Rotation about a fixed axis: It is a motion in which all particles of the rigid body move in circular paths about the axis of rotation.

	Vector Approach	Direction & Sense	Scalar Approach
Velocity of the particle	$\bar{v} = \bar{r} = \bar{\omega} \times \bar{r}$	Perpendicular to radius	$v = \omega . r$
Normal or Radial acceleration	$\bar{f}_n = \bar{\omega} \times (\bar{\omega} \times \bar{r})$	Along the radius and towards fixed axis	$f_n = \omega^2 r = \frac{v^2}{r} = \omega v$
Tangential acceleration	$\bar{f}_t = \bar{\alpha} \times \bar{r}$	Perpendicular to radius	$f_t = \alpha . r$
Total acceleration	$\bar{f} = \bar{f}_n + \bar{f}_t$		

For uniform rotation: $\theta = \theta_0 + \omega . t$

For uniform acceleration rotation: $\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha . t^2$

$$\omega = \omega_0 + \alpha . t$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

General Plane motion: It is the combination of translation and rotation.

(a) Absolute motion analysis is the approach in which the geometric relationship that define the configuration of the body involved is generally used. In order to obtain velocity and acceleration, time derivatives of the defining geometric relationship are taken into consideration.

(b) Relative motion analysis is an approach in which combination of translation and rotations are used for both velocity and acceleration.

$\overline{v_B} = \overline{v_A} + \overline{v_{B/A}}$ and $\overline{f_B} = \overline{f_A} + \bar{f} + (\bar{f}_{A/B})_n + (\bar{f}_{B/A})_t$

$v_{B/A} = \omega \times AB$. Perpendicular to (AB). $(f_{B/A})_n = \omega^2 \times (AB)$, along (AB) & towards A.

$(f_{B/A})_t = \alpha \times (AB)$. Perpendicular to (AB)

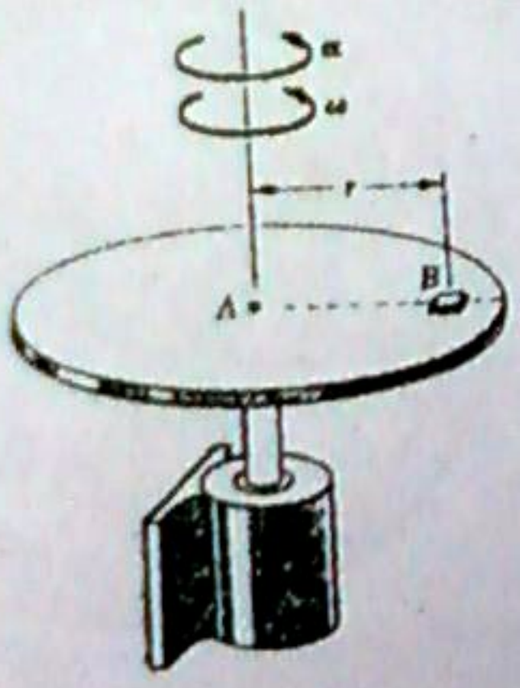
Problems:

1. A small grinding wheel is attached to the shaft of an electric motor which has a rated speed of 3600 r/min. When the power is turned on, the unit reaches its rated speed in 5 s, and when the power is turned off, the unit coasts to rest in 70s. Assuming uniformly accelerated motion, determine the number of revolutions that the motor executes (a) in reaching its rated speed, (b) in coasting to rest.

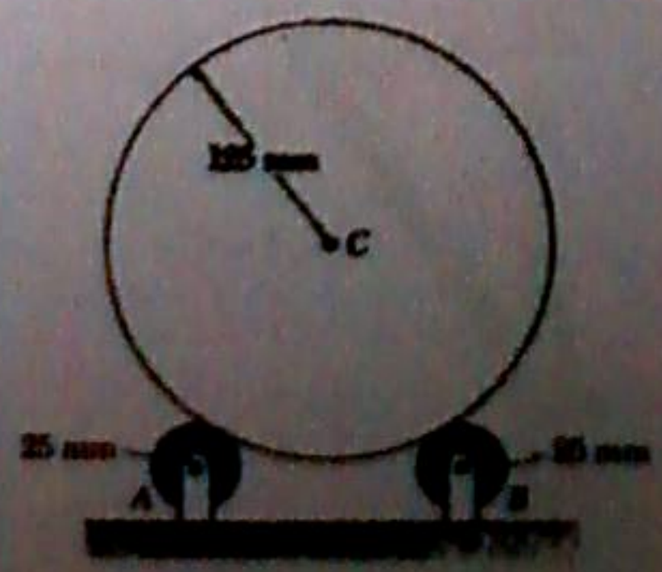
2. As steam is slowly injected into a turbine, the angular acceleration of the rotor is observed to increase linearly with the time t. Knowing that the rotor starts from rest at t = 0 and that after 10s the rotor has completed 20 revolutions, write the equations of motion for the rotor and determine (a) the angular velocity at t = 20s, (b) the time required for the rotor to complete its first 40 revolutions.

3. The earth makes one complete revolution on its axis in 23.93h. Knowing that the mean readius of the earth is 6370km, determine the linear velocity and acceleration of a point on the surface of the earth (a) at the equator, (b) at Philadelphia, latitude 40^0 north, (c) at the North Pole.

4. It is known that the static-friction forces between the small block B and the plate will be exceeded and that the blocks will stat sliding on the plate when the total acceleration of the block reaches 4 m/s^2. If the plate starts from rest at t = 0 and is accelerated at the constant rate of 5 rad/s^2, determine the time t and the angular velocity of the plate when the block starts sliding, assuming r= 250 mm.



5. A mixing drum of 125-mm outside radius rests on two casters, each of 25-mm radius. The drum executes 15 revolutions during the time interval t, while its angular velocity is being increased uniformly from 20 to 50 r/min. Knowing that no slipping occurs between the drum and the casters, determine (a) the angular acceleration of the casters, (b) the time interval t.



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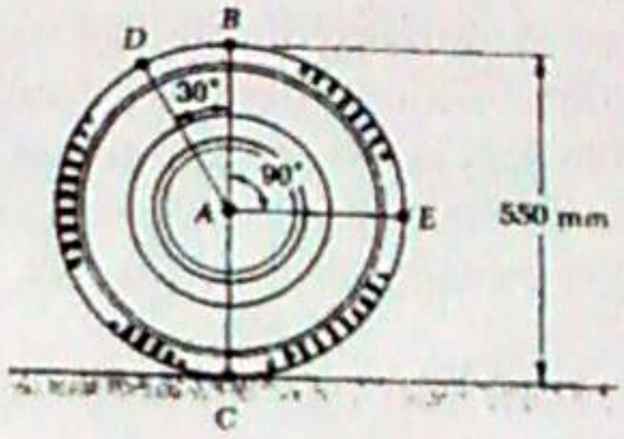
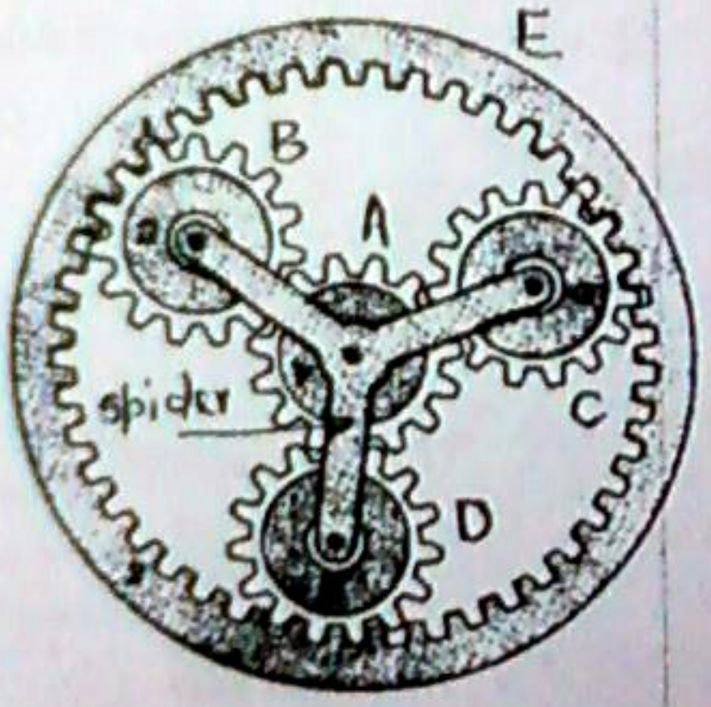
6. An automobile travels to the right at a constant speed of 90 km/h. If the diameter of a wheel is 550 mm, determine the velocities and accelerations of points B,C,D, and E on the rim of the wheel.



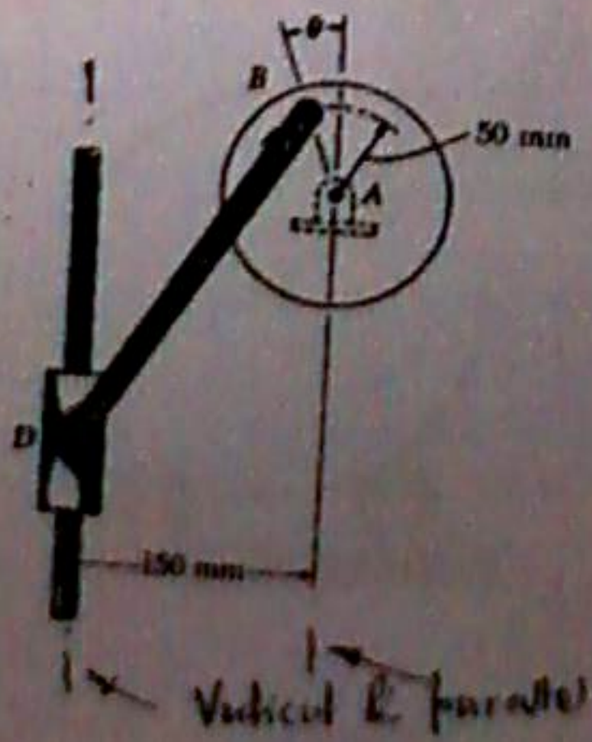
7.

(a) In the planetary gear system shown, the radius of gears A,B,C, and D is 'a' and at the radius of the outer gear E is '3a'. Knowing that the angular velocity of gear A is ϖ_A clockwise and that the outer gear E is stationary, determine (i) the angular velocity of each planetary gear, (ii) the angular velocity of the spider connecting the planetary gears.

(b) In the planetary gear system shown, the radius of gears A,B,C and D is 30mm and the radius of the outer gear E is 90 mm. Knowing that gear E has an angular velocity of 180 r/min clockwise and that the central gear A has an angular velocity of 240 r/min clockwise, determine (i) the angular velocity of each planetary gear, (ii) the angular velocity of the spider connecting the planetary gears.



8. The disk shown has a constant angular velocity of 500 r/min counter-clockwise. Knowing that rod BD is 250 mm long, determine the angular velocity and acceleration of rod BD also the velocity and acceleration of collar D when (a) $\theta = 0^0$ (b) $\theta = 90^0$.



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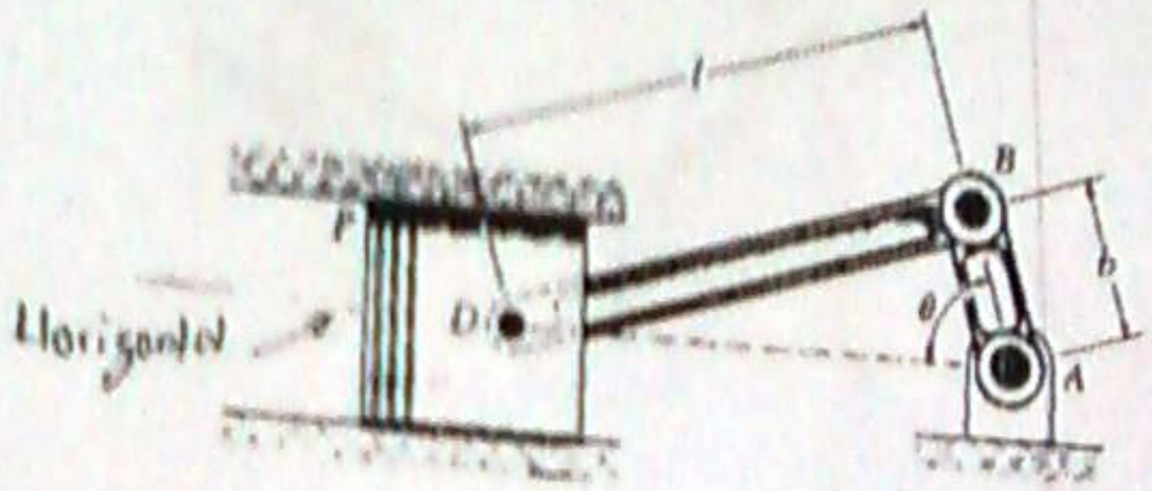
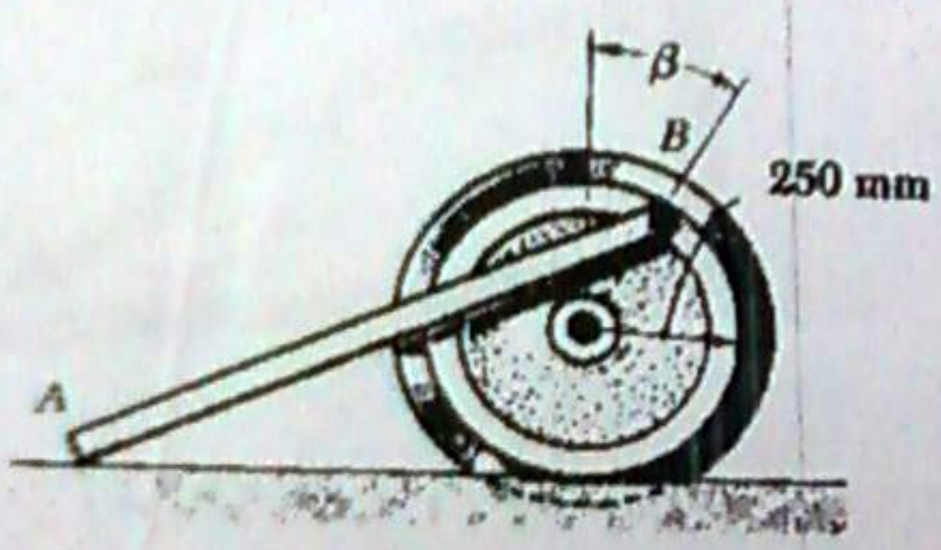
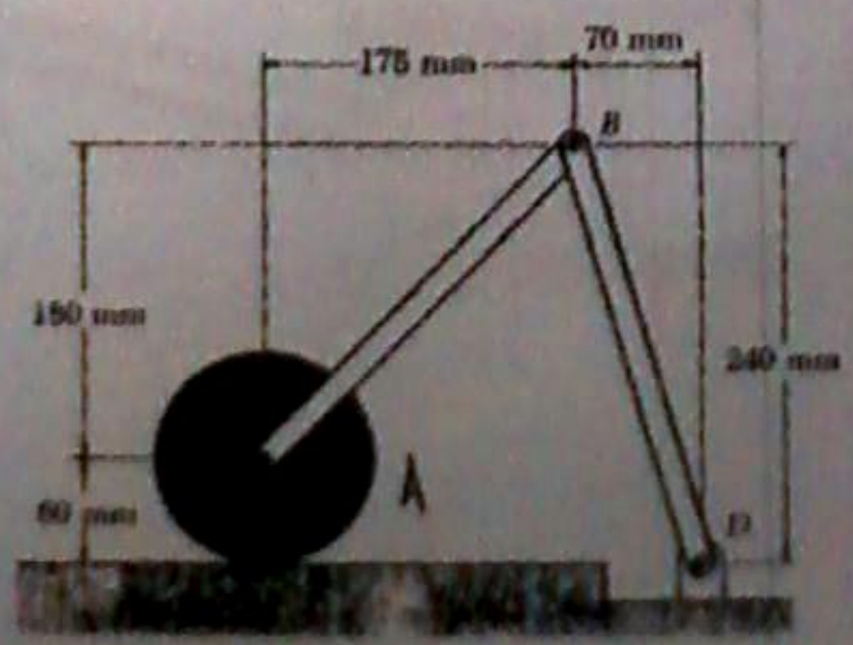
9. In the engine system shown, l= 200mm and b = 75 mm; crank AB rotates with a constant angular velocity of 2000 r/min clockwise. Determine the velocity and acceleration of piston P as well as the angular velocity and acceleration of the connecting rod for the position corresponding to (a) $\theta = 0$, (b) $\theta = 90^0$, (c) $\theta = 180^0$.



10. The flanged wheel shown, rolls to the right with a constant velocity of 1.5 m/s. knowing that rod AB is 0.9 m long. Determine the velocity of A and the angular velocity of the rod when (a) $\beta = 0$, (b) $\beta = 90^0$.



11. A 60-mm-radius wheel is connected to a fixed support D by two links AB and BD. At the instant shown, the velocity of the center A of the wheel is 300 mm/s to the left. Determine (a) the angular velocity of each link, (b) the velocity of pin B.



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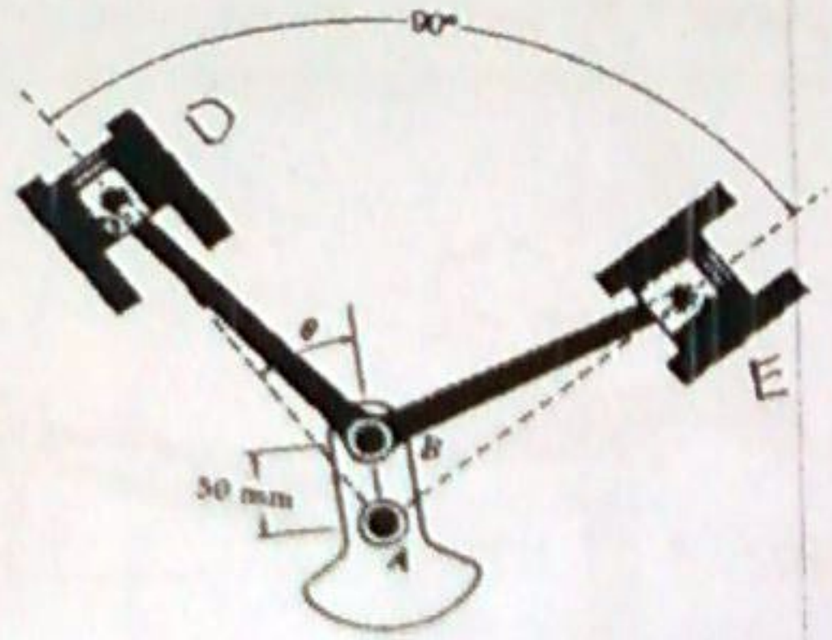
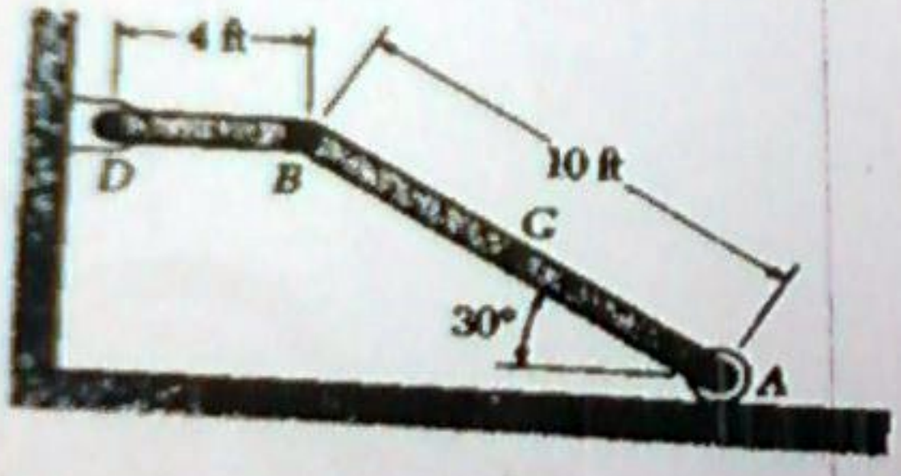
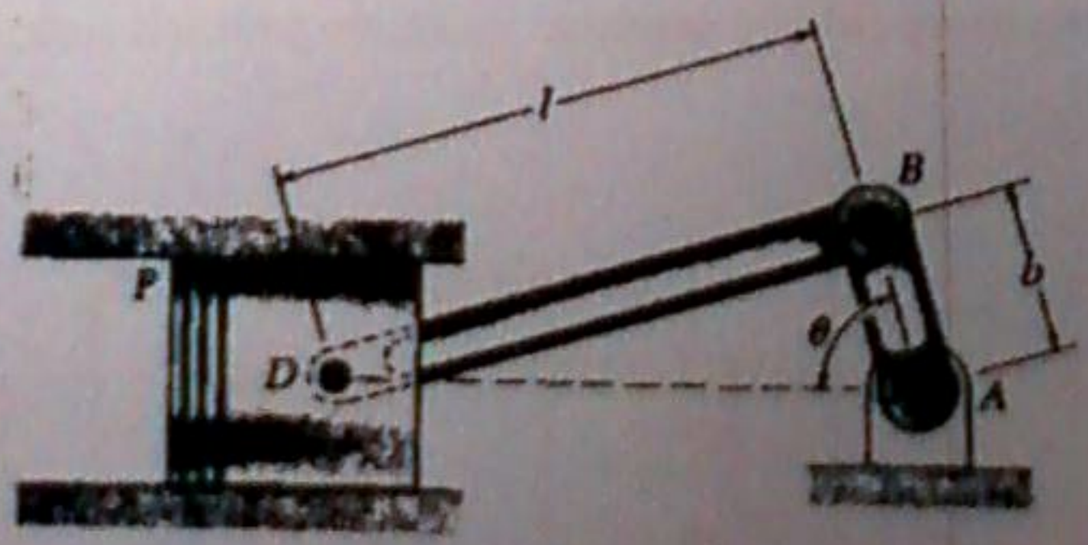
12. In the two-cylinder air compressor shown the connecting rods BD and BE are each 188 mm long and crank AB rotates about the fixed point A with a constant angular velocity of 1500 r/min clockwise. Determine the acceleration of each piston when $\theta = 0$.



13. In the position shown, end A of rod AB has a velocity of 0.75 m/s and an acceleration of 0.54 m/s², both directed to the left. Determine (a) the angular acceleration of rod AB, (b) the acceleration of the midpoint G of rod AB.



14. Crank AB rotates with a constant clockwise angular velocity ϖ, and $\theta = 0$ at $t = 0$. Using analytical method, derive an expression for the velocity and acceleration of piston P in terms of the time t.



15. The position of rod AB is controlled by a disk of radius r which is attached to yoke CD. Knowing that the yoke moves vertically upward with a constant velocity V_0, derive an expression for the angular velocity of rod AB.